REDISTRIBUTION AND INSURANCE: MANDATORY ANNUITIZATION WITH MORTALITY HETEROGENEITY

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Abstract

This paper examines the distributional implications of mandatory longevity insurance when there is mortality heterogeneity in the population. Previous research has demonstrated the significant financial redistribution that occurs under alternative annuity programs in the presence of differential mortality across groups. This paper embeds that analysis into a life cycle framework that allows for an examination of distributional effects on a utility-adjusted basis. It finds that the degree of redistribution that occurs from the introduction of a mandatory annuity program is substantially lower on a utility-adjusted basis than when evaluated on a purely financial basis. Complete annuitization is shown to be optimal even when annuities are not actuarially fair for each individual, so long as there are no administrative costs and no bequest motives. In the presence of bequest motives, mandatory complete annuitization can make some individuals worse off. Even with strong bequest motives, however, welfare can be substantially enhanced by annuities by allowing individuals to only partially annuitize their wealth. Finally, life annuities with "period certain" bequest options are shown to be inferior to partial annuitization, while having identical distributional effects.

Most public pension systems combine elements of redistribution and insurance. For example, the U.S. Old Age Survivors Insurance program (OASI) uses a non-linear benefit formula that provides a higher replacement rate for lower income workers in an effort to make the system progressively redistributive. At the same time, OASI insures individuals against longevity risk through the provision of benefits in the form of life annuities.

For some types of risk, providing insurance and engaging in progressive redistribution are complementary activities. This is true, for example, with Disability Insurance. In the U.S., workers covered by the DI program are provided with insurance against income loss in the event of becoming disabled. Because individuals who are disabled have, by definition, diminished earnings capacity, this same program serves a progressively redistributive role. Even on an *ex ante* basis, if lower wage individuals have a higher probability of becoming disabled, then a disability insurance program would even redistribute from higher to lower income individuals in expectation.

For other types of risk, however, the provision of insurance can have regressive distributional effects. Longevity risk is one such case. In a life-cycle setting, individuals who do not know how long they will live are, in general, made better off by annuitizing their wealth. However, because high-income individuals have longer life expectancies, they will have a higher expected present value of annuity payments than will low income individuals, if everyone is required to annuitize at a uniform price as in most public pension plans.

To a large extent, these effects have been considered separately in the literature on Social Security and annuitization. There is a large literature examining the within-generation distributional effects of existing Social Security system in the United States (e.g., Gustman & Steinmeier 2000, Liebman 2000, Coronado, Fullerton & Glass 2000), and a smaller but growing literature examining redistribution within an individual accounts system (e.g., Brown 2000, Feldstein & Liebman 2000). Most of these analyses, however, have focused on purely financial measures of redistribution, such as internal rates of return, money's worth, dollar transfers, and implicit tax rates. While these financial calculations are informative and interesting in their own right, they implicitly ignore the value of any insurance that is provided by the pension system under consideration.

Another strand of literature has focused on measuring the insurance value of annuitization for representative life-cycle consumers (e.g., Mitchell, et al 1999, Brown 2001). These papers generally quantify the utility gains from access to actuarially fair annuity markets by finding how much incremental, non-annuitized wealth would be equivalent to providing access to an actuarially fair annuity market (sometimes called the "annuity equivalent wealth"). A standard result from this approach is that a 65-year old male with log utility, whose mortality expectations mirror that of the population average, would find annuities equivalent in utility terms to a 50% increase in wealth. With few exceptions, however, these utility-based calculations have been conducted only for "average" consumers who have access to annuities that are actuarially fair, i.e., that are priced using the individual specific mortality rates. Little has been done to examine the utility implications of annuitizing in an environment of heterogeneous mortality.

The contribution of this paper is to unite these two strands of literature by examining the distributional impact of alternative annuity designs in a framework that incorporates the utility value of the longevity insurance. In particular, it examines how the annuity equivalent wealth varies across socioeconomic groups when annuities are priced uniformly. Staying with the no bequest assumption, this approach provides answers to three types of questions. First, under what conditions are individuals, particularly those in high-mortality risk groups, made better off by annuitizing at a uniform price? Second, how much redistribution is there on a utility-adjusted basis? Third, how are the answers to the first two questions affected by alternative annuity designs? For example, would individuals with shorter life expectancies prefer constant real annuities or some other path of payments? This paper then further extends the previous literature on annuity valuation by incorporating an explicit bequest motive into the utility function. This allows one to calculate the value of the annuity under alternative assumptions about the extent to which payments to heirs are permitted in the system, including partial annuitization and period certain bequest options.

This approach yields five interesting findings. First, in the absence of pure administrative costs, uniform priced annuities can make all life-cycle consumers better off, even those with mortality rates that are substantially higher than those used to price the annuity. Second, the amount of redistribution that arises from mandatory annuitization is much smaller on a utility adjusted basis than on a financial basis. Third, in the presence of mortality heterogeneity, even high mortality risk individuals are generally made better off by providing annuities that are close to constant in real terms. Fourth, while complete annuitization can actually make some individuals with bequest motives worse off, a system that requires only partial annuitization, and thus leaves the individual with some bequeathable wealth, still achieves substantial utility gains. Fifth, this paper demonstrates that annuities with period certain options are dominated by a portfolio allocation that allows the individual to partially annuitize.

This paper proceeds as follows. Section 1 provides a review of the literature on why annuities are valuable to representative retirees. Section 2 presents evidence on the interaction between mortality and socioeconomic status using data from the National Longitudinal Mortality Study, and discusses the impact of this on financial measures of distribution. Section 3 uses a simplified, two-period model to provide intuition for how the utility value of an annuity is affected by differential mortality. Section 4 discusses the dynamic programming methodology for solving for annuity valuation in a multi-period problem with liquidity constraints. Section 5 reports dynamic programming simulation results of the annuity equivalent wealth for multi-period life cycle individuals with more realistic constraints on annuity payments. Section 6 incorporates a bequest motive into the simulation model and examines full and partial annuitization, as well as period certain guarantees. Section 7 concludes.

1. The Insurance Value of Annuitization

In a widely cited article, Yaari (1965) demonstrated that a risk averse, life-cycle consumer facing an uncertain date of death would find actuarially fair annuities of substantial value. In fact, under certain conditions, including the absence of bequests and the absence of other sources of uncertainty, life cycle consumers find it optimal to invest 100% of wealth into actuarial notes. More recent theoretical work indicates that annuities are often welfare enhancing in a broader set of cases than those allowed by Yaari, including in the presence of aggregate risk, adverse selection, and intertemporal non-additivity of the utility function (Brown, Davidoff, and Diamond 2001). Other extensions, such as allowing for precautionary savings and bequest motives, tend to reduce the value of annuitization.

Annuities derive their value from the elimination of longevity risk. In the absence of annuities, individuals facing an unknown date of death must allocate their wealth across an uncertain number of periods. Unless the individual lives to the maximum lifespan, following the optimal consumption path will result in the individual dying with positive financial wealth. Assuming the individual does not value bequests, the individual would have been better off, *ex post*, had she consumed more each period while alive. *Ex ante*, however, following a more aggressive consumption path would have exposed her to the risk of having very low consumption levels in the event that she lived longer than expected. This problem arises in the absence of annuities because the individual is unable to allocate wealth in a state contingent manner. Instead, she must, for any given future period, set aside an equal amount of wealth for the state in which she is alive, and thus values consumption, and the state in which she is dead and does not value consumption.

Annuities partially complete the market by allowing an individual to make future resources survival state contingent. In particular, annuities allow the individual to increase the income available in future periods conditional on being alive, in return for accepting zero resources in the event that she dies. If an individual has no bequest motive and therefore cares only about future states in which she is alive, this enables her to consume more each period while alive and completely eliminate the risk of living "too long" with resources insufficient to support desired consumption levels.

An equivalent way to view the opening up of an annuity market for an individual without bequest motives is to think of it as a change in the price of future consumption. In a non-annuitized world, the period 0 price of consumption in period t is $(1+r)^{-t}$, meaning that a person must give up $\$(1+r)^{-t}$ in period 0 to obtain \$1 of consumption in period t. If actuarially fair annuities are available, and the probability of surviving from period 0 to period t is P, then the price of period t consumption becomes $P(1+r)^{-t}$. Because $P<1$, this is equivalent to a reduction in the price of future consumption relative to the world with no annuities. As such, the individual's consumption opportunity set is expanded, leading to a higher level of utility.

How large are these utility gains? Previous work indicates that they can be quite large. For example, a 65-year old man with mortality expectations based on 1998 average U.S. population life tables suggests that, with log utility, obtaining access to an actuarially fair real annuity market is equivalent to a 50% increase in wealth (Brown, Mitchell & Poterba, 2001). However, all of these studies have assumed that individuals have access to annuity markets that are actuarially fair, i.e., that the annuity is priced according to each individual's own mortality rates. In most realistic policy settings, such as public or private pension systems, individuals with heterogeneous mortality are pooled into a common annuity market. As such, very few individuals have access to annuities that are priced in a manner that is actuarially fair at the individual level, even if the system is actuarially fair on average. As such, the utility gains from annuitization in such a setting will vary across individuals.

2. <u>Mortality Heterogeneity and Annuity Prices</u>

There is substantial heterogeneity in expected lifetimes in the U.S. population. In addition to differences by age and gender, it has been substantially documented that mortality rates are correlated with race (Preston et al 1996), ethnicity (Sorlie, et al 1993), income (Deaton

& Paxson 1999), wealth (Attanasio & Hoynes 2000), marital status (Brown & Poterba 2001), and educational attainment (Kitawaga & Hauser 1972, Deaton & Paxson 1999, Lantz et al 1998). In general, these correlations work in the direction that individuals of higher socioeconomic status live longer than those in lower socioeconomic groups. For example, whites live longer than blacks, higher income and higher wealth individuals live longer than individuals with less wealth, married people live longer than singles, and more highly educated individuals live longer than less educated individuals. There is also controversial evidence suggesting that Hispanics live longer than whites in the U.S., though this appears to be more true for foreign born than U.S. born Hispanics, and may be due to data contamination.

These mortality differentials can have significant effects on the progressivity of mandatory annuitization schemes, including most public pension systems. For example, several recent papers have suggested that the progressivity of the OASI benefit formula, which provides a higher replacement rate for lower wage individuals, is largely offset by the fact that higher income individuals tend to live longer than lower income individuals (Liebman 2000, Coronado, Fullerton, & Glass 2000). The result is that, on net, there is very little progressivity in the existing US Social Security system.

As many countries, including the United States, consider proposals to supplement or partially replace the existing pay-as-you-go, defined benefit system with a system of individual accounts, there have been concerns raised about the distributional implications. In particular, if there is no redistribution done in the accumulation phase, such as through a system of taxes and/or subsidies on contributions, then there will be no offset of any distributional effects that arise from mortality heterogeneity in the payout phase. For example, if all individuals were required to annuitize their retirement accounts at a uniform price upon reaching age 67, the expected present value of future annuity payments would be substantially smaller for individuals with higher mortality probabilities, even if the account balances were identical in size. This would result in substantial dispersion in the money's worth of annuities across different individuals and demographic groups.

How large are these effects? To answer this, it is necessary to construct a set of mortality tables that are differentiated based on demographic characteristics. This paper will use mortality estimates that are differentiated by age, gender, race, ethnicity, and educational attainment. The group specific mortality differentials are estimated using data from the National Longitudinal Mortality Study (NLMS). The NLMS is a survey of individuals who were originally included in the Current Population Survey and/or Census in the late 1970s and early 1980s. Throughout the 1980s, death certificate information from the National Death Index was merged back into the survey data, allowing researchers to compare the death rates of individuals on the basis of demographic characteristics at the time of the interview.

Age specific mortality rates are constructed from the NLMS for black, white and Hispanics males and females, a total of six groups. The white and black groups are then further differentiated based on education, less than high school, high school plus up to three years of college, and college graduates. Due to small sample sizes, it is not possible to differentiate Hispanics along educational lines.

Several steps are required to turn these NLMS estimates into cohort mortality tables for specific groups. First, the NLMS sample is split into groups based on the gender, race, ethnic, and educational categories. For each group g, the age-specific, non-parametric (np) mortality rate, $q^{np}_{x,g}$, is calculated as the fraction of those individuals age x who die before attaining age x+1. This procedure provides a simple, non-parametric estimate of the age specific mortality rate for individuals with the characteristics of group g.

In order to correct for non-monotonicity that occasionally arises due to small cell sizes in some populations, the non-parametric estimates, $q^{np}_{x,g}$, are treated as the independent variable in a non-linear least squares regression on age x. The non-linear regression is used to estimate three parameters of a Gompertz/Makeham survival function. As explained in Jordan (1991), with the proper choice of the three parameters, this formula can be applied from about age 20 almost to the end of life. The Gompertz/Makeham formula used is:

$$l_x = ks^x g^{c^x} \tag{1}$$

$$\text{where } k = \frac{l_0}{g} \text{ and } q_x = \frac{l_{x+1} - l_x}{l_x}$$

x is age, and g, c, and s are the parameters to be estimated. Note that if l_0 is set equal to one, then l_x is simply the cumulative survival probability to age x. Using the NLLS estimates of g, c, and s, one then has a "Makeham formula" that gives mortality q_x as a function of x. Let us denote these fitted values of mortality for group g at age x as $q^{fit}_{x,g}$. An important feature of this approach is that fitted mortality rates are a monotonically increasing function of age x. Another feature is that it allows one to create out-of-sample estimates of mortality. Therefore, while only data from age 25 to 84 is used to fit the curve, the formula can provide estimates of mortality for ages outside of this range.

Once these predicted mortality rates are in hand, the next step is to convert them into cohort life tables for each group. This requires two related assumptions. The first is that the ratios of a group's age-specific mortality to that of the population as a whole ($q_{x,g}/q_x$) in the NLMS sample is the an accurate portrayal of these ratios in the full population in 1980. The second assumption is that these ratios are constant over time. By invoking these two assumptions, it is possible to then construct a group specific cohort life tables for any year.

Specifically, let $q^{fit}_{x,g}$ be the fitted value of the mortality rate for an individual age x belonging to group g, and let q^{fit}_{x} be the mortality rate for an individual age x for the population as a whole, both from the fitted NLMS data. Let q^{SSA}_{x} be the age-specific mortality rate from the 1978 birth cohort table from the Social Security Administration, which represents individuals turning age 22 in the year 2000 (the group of study in this paper). Then the cohort, group specific mortality rates are constructed as follows:

$$q_{x,g}^{SSA} = q_x^{SSA} \frac{q_{x,g}^{fit}}{q_x^{fit}} \qquad (2)$$

The one exception to this methodology is that in the case of college educated black males and females, the mortality ratio between college and high school is assumed to be the same for blacks as for whites. This ratio is applied to the fitted q's for blacks with a high school education in order to construct the estimate for a college educated black. This was done because the sample sizes at many ages were too small for college-educated blacks to reliably construct an independent estimate.

Table 1 reports how the life expectancy of a 22 year old in the year 2000 varies by the gender, race, ethnicity, and education as calculated using the above methods. The average 22 year old male can expect to live to age 77.4, while the average 22 year old woman can expect to live to age 83.4. However, these estimates vary widely by race. White, black, and Hispanic 22-year old males have life expectancies of 78.1, 71.4 and 77.5 years respectively, while white, black and Hispanic females have life expectancies of 83.8, 79.7, and 85.6 years respectively. Life expectancy at age 22 also varies substantially by education level. White men with less than a high school education have a life expectancy at age 22 of 75.8 years, a full 5.2 years less than that of a white male with a college degree. Low educated black males have by far the lowest age 22 life expectancy of any group examined, at 67.9 years. The highest life expectancy is college educated white women, who can expect to live to age 86.3.

Table 1 also reports the life expectancy as of age 67 for this same cohort. As can be seen, there is still a substantial range in the estimates, although the differential in years is not as large as at age 22. This is because much of the life expectancy difference that arises for 22 year olds is due to higher mortality probabilities in the pre-retirement period. Conditional on reaching age 67, these differences are diminished. The numbers suggest that a 67 year-old white can expect to live approximately 16 months longer than a 67 year old black. When further differentiating by educational attainment, the difference is naturally larger, with a 3.4 year difference between college educated white men and less than high school educated black men.

One can use these mortality estimates to construct a "money's worth" of an annuity that is priced based on the average mortality in the population. A money's worth measure is simply the expected present value of annuity payments per dollar spent to purchase the annuity, and has been used in many past studies of annuity prices (Friedman & Warshawsky 1988 and 1990, Warshawsky 1988, Mitchell et al 1999). Table 2 reports the money's worth ratio for the cohort entering the workforce in the year 2000. This purely financial measure indicates that the money's worth of an inflation indexed life annuity for a 65 year old black male with less than a high school education would be only 0.800, while a white woman with a college education would have a money's worth of 1.106. Viewed solely from this financial perspective, mandating annuitization at a uniform price is tantamount to a system of taxes and transfers that takes resources from poorly educated black men and gives it to highly educated white woman. This is due to the fact that an annuity, by design, serves to transfer resources from shorter-lived to longer-lived individuals, combined with the fact that there is heterogeneous mortality in the population.

Brown (2000) explores the money's worth of a richer set of annuity options, and finds that the dispersion in money's worth across groups can be substantially reduced by considering annuities that "front-load" annuity payments or offer bequest options. As reported in Table 2, an annuity that declines in real value by 3% per year increases the money's worth for low educated black men to 0.83. Even more striking, offering an annuity with a 20-year period certain guarantee increases the money's worth to black men with less than a high school education to 0.955, as indicated in column 3.

The money's worth, however, is purely a financial measure, and as such it ignores the insurance value that individuals derive from the elimination of longevity risk. To assess the welfare effect of differential mortality, it is necessary to embed the heterogeneous mortality into a utility-based model.

3. <u>Annuity Valuation with Heterogeneous Populations</u>

Previous studies have used an "annuity equivalent wealth" measure to quantify the gains from actuarially fair annuitization. To understand this approach analytically, it is useful to examine a much simplified problem. Consider a two-period model for a single consumer, with additively separable log utility of consumption, and the interest rate and time preference rate both equal to zero. Let P be the probability that the individual will survive to period 2, and let ϕ be the period 1 price of consumption in period 2. Then the consumer's problem is:

$$\underset{\{C_1, C_2\}}{Max} \ \ln C_1 + P \ln C_2 \tag{3}$$

subject to:

$$C_1 + fC_2 = W \tag{4}$$

Taking first order conditions, we find that

$$C_2 = \frac{P}{f} C_1 \tag{5}$$

Note that if no annuities are available, ϕ=1, and the optimal consumption path is declining proportionally with the probability of survival. If annuities are actuarially fair, then ϕ=P, and the individual wishes to perfectly smooth consumption over the life cycle. If annuities are available, but are more expensive than actuarially fair, then 1>ϕ>P, and consumption will decline at an intermediate rate.

Solving (2) and (3) and plugging into (1), we find that the indirect utility function V(P,ϕ,W) is:

$$V(P, f, W) = (1+P)\ln\left(\frac{W}{1+P}\right) + P\ln\left(\frac{P}{f}\right) \tag{6}$$

Denote the Annuity Equivalent Wealth as α, which is implicitly defined as:

$$V(P,1,\alpha W) = V(P,\phi,W) \tag{7}$$

The left hand side of equation 5 is the utility level achieved when the individual does not have access to annuities, so that the price of second period consumption is equal to one, but has additional wealth. The right hand side of equation 5 is the utility level achieved when an individual has access to an annuity with a price of ϕ. The Annuity Equivalent Wealth, α, is a measure of the additional wealth that must be given to the individual in the absence of annuities to be as well off as if the individual could annuitize at a price of ϕ.

Solving equation (5), we find that $\alpha = \phi^{-\left(\frac{P}{1-P}\right)}$. Naturally, when no annuities are available, ϕ=1 so that α=1. When the annuity is actuarially fair, ϕ=P, and $\alpha = P^{-\left(\frac{P}{1-P}\right)}$. For example, if ϕ=P=.5, then α=1.26, indicating that an individual would be indifferent between \$1.26 of non-annuitized wealth, and \$1.00 of annuitized wealth. Therefore, access to actuarially fair annuity markets can be said to be worth a 26% increase in wealth.

This highly simplified framework allows one to immediately see several stylized results. Result 1: $\alpha>0$ for all $0<P<1$ and $0<\phi<1$. So long as there are no additional administrative costs of annuitization, all consumers are made better off by annuitization. Importantly, even if individuals with very short life expectancies are required to annuitize in a market where pricing is based on high survival probabilities (i.e., low P and high ϕ), the annuity equivalent wealth exceeds 1.0. Thus, the original Yaari (1965) assumption that annuities be priced actuarially fair is overly restrictive. The intuition for this is straightforward – with no loading costs, the availability of annuities that are not actuarially fair for an individual still have the effect of reducing the price of future consumption for that individual, and thus making the consumer better off. (For a more comprehensive extension of Yaari's theory, see Brown, Davidoff and Diamond 2001).

Result 2: For fixed P (0<P<1), $\frac{\partial a}{\partial f} < 0$. Any individual with an uncertain lifespan values annuities more highly when they are priced using lower survival probabilities. This is quite intuitive, since it simply states that all individuals are better off when the price of future consumption falls.

Result 3: For fixed ϕ, $\frac{\partial a}{\partial P} > 0$. For a fixed price, increasing an individual's survival probability makes the annuity more valuable because they are more likely to survive to consume the annuity.

Result 4: When P=ϕ, $\frac{\partial a}{\partial P} > 0$ up to some $\overline{P}$, and then $\frac{\partial a}{\partial P} < 0$. When annuities are actuarially fair for each individual, increasing P from 0 to 1 has a positive and then negative effect on annuity valuation. This is because a change in P now has two effects. First, individuals with longer life expectancies (higher P) are more likely to survive to the second period and thus consume the annuity, which makes annuities more valuable. Second, high P individuals must pay more for second period consumption, which makes annuitization less valuable. These effects work in opposite directions, and thus a plot of α against P is hump shaped. In this simple example given here, the value of annuitization peaks at P=.278, a relatively low survival probability. If we were to compare a cross-section of individuals, all of whom had a P>.278, we would find that *individuals with shorter life expectancies value individually priced annuities more highly than those with longer life expectancies*. This is in contrast to the intuition of result 2, which is the standard intuition about annuity valuation. The reason for this result is that individuals with lower survival rates are rewarded with a lower price of consumption for the second period.

In this two-period problem, P is a sufficient statistic for both the life expectancy (e.g., the mean) and the degree of longevity risk (e.g., higher moments). When one moves to a multi-period problem, life expectancy is no longer a sufficient statistic for how much mortality risk one faces, and therefore, for how much one values an annuity. Life expectancy is an average, and the simple economics of risk suggests that the degree of uncertainty around the mean also matters. In fact, it is possible for a person with a longer life expectancy to value an identical annuity less than someone with a shorter life expectancy. For a trivial (and very hypothetical) example, consider a 65 year-old man who knows he will live exactly 20 more years and die on his 85th birthday. This person has no risk to insure against, and the annuity is worth no more to him than the simple discounted value of the 20 years of payments. If a second man has an identical 20-year life expectancy, but substantial risk around this mean, he will value the annuity more highly. As such, there exists some $\varepsilon>0$ such that we can reduce this second person's life expectancy to 20-ε, and still have a higher annuity equivalent wealth than the person who will live to 20 for sure, due to the uncertainty around this mean. As such, it is not really accurate to claim that an individual with a longer life expectancy will value annuities more highly. A multi-period model will be discussed in the next section.

So far, we have been assuming that there are no mark-ups of price over marginal cost. If there are mark-ups, such as in the form of administrative costs, the budget constraint in (4) can be rewritten as:

$$C_1 + q \cdot f \cdot C_2 = W \tag{8}$$

where $\theta>1$ for a positive mark-up. This leads us to another straightforward result:

<u>Result 5</u>: If $(\theta\phi)>1$, then $\alpha<1$. If administrative costs are high enough to completely offset the price reduction that arises from the mortality rates used to price the annuity, then this has the effect of making second period consumption more expensive, and annuities become less valuable.

This two-period model is useful insofar as it builds some simple intuition for how the value of annuitization is related to the price of annuities, survival probabilities and administrative costs. These results generally apply to multi-period models in which the individual has the ability to choose survival contingent consumption in each period separately, i.e., Arrow-Debreu markets are complete. For example, if an individual finds that his own survival probabilities in some periods are higher than those used in the pricing of annuities, he would choose to consume more in those states. In most "real world" annuity markets, however, the structure of annuity payments typically constrains one's ability to do this. For example, in the U.S. Social Security system, individuals are forced to annuitize in a constant real annuity, and are constrained against borrowing from future annuity payments. They must therefore purchase units of consumption across periods in fixed proportions, and this means they are unable to precisely match the annuity to their preferred consumption profile. When these constraints bind, this will have the effect of reducing annuity value. Importantly, these constraints will have differential effects on groups with different mortality expectations. Therefore, the differential utility impact of multi-period annuity contracts needs to be examined in a multi-period setting to determine if the basic results of the two-period model go through.

4. Multi-Period Annuity Valuation

Calculating the Annuity Equivalent Wealth (α) for more realistic, multi-period settings with constraints on the annuity payments can in some cases be solved in closed form. Generally, however, the presence of liquidity constraints imposed by the annuity structure makes closed form solutions difficult to obtain. In such cases, one way to solve for the α is to use dynamic programming techniques.

To generalize the problem, let $U(C_t)$ represent the one-period utility function defined over real consumption, ρ the utility discount rate, and T the maximum possible life-span of an individual. Then the consumer's problem, assuming additive separability over time, is:

$$Max_{\{C_t\}}\left[\sum_{t=1}^{T-age+1}\frac{P_t \cdot U\left(C_t\right)}{\left(1+r\right)^t}\right] \tag{9}$$

where P_t is the probability of surviving to period t, subject to the following constraints:

$$\begin{aligned} &(i)\ W_0 \quad given \\ &(ii)\ W_t \geq\ 0,\ \forall t \\ &(iii)\ W_{t+1}\ =\ (W_t - C_t + A_t)(1+r) \end{aligned} \tag{10}$$

In these constraints, W_t is non-annuitized wealth in period t, C_t is consumption, and A_t is the annuity payment that can be purchased when annuity markets are available. Assume that the individual, prior to any annuitization, has financial wealth W^*. Then for the case in which no annuities are available, $W_0 = W^*$, and $A_t=0$, $\forall$t. In the case in which the individual fully annuitizes all financial assets, then $W_0 = 0$, and A_t is determined by the pricing in the annuity market. For the special case in which the annuity is actuarially fair for the individual, A_t is determined by the equation:

$$W^* = \sum_{t=1}^{T-age+1} \frac{A_t \cdot P_t}{(1+r)^t (1+p)^t} \tag{11}$$

In equation (8), the real interest rate is represented by r, and the inflation rate by π. Note that this formula determines the nominal value of a fixed nominal annuity. The real value of this annuity declines by the factor $1/(1+\pi)$ each period. By setting $\pi=0$, equation (3) can be used to determine the starting value of a real annuity as well. In the simulations that follow, it will be assumed that $r=\rho=.03$. By replacing the individual's P_j with the average P_j for the annuitizing population, we can construct the annuity payments available in a uniform price system. It is also straightforward to incorporate administrative loading costs into the calculation by multiplying the right-hand side of equation 11 by one minus the load factor.

In order to use dynamic programming techniques to solve for the optimal consumption path, it is useful to introduce a value function $V_t(W_t)$, which is defined as:

$$V_t(W_t) = Max_{\{C_t\}} \left[\sum_{t=1}^{T-age+1} \frac{P_t U(C_t)}{(1+r)^t} \right] \tag{12}$$

subject to the constraints in equation (10).

The value function at time t is the present discounted value of expected utility evaluated along the optimal path. This value function satisfies the following recursive Bellman equation:

$$Max_{\{C_t\}} V_t(W_t) = Max_{\{C_t\}} U(C_t) + \frac{(1-q_{t+1})}{(1+r)} V_{t+1}(W_{t+1}) \tag{13}$$

where q_{t+1} is the one period mortality probability, i.e., the probability of dying in period t+1 conditional on surviving through period t. The relationship between q and P is:

$$P_t = \prod_{j=1}^{t} (1-q_j) \tag{14}$$

The Bellman equation reduces the full maximization problem to a series of 2-period problems that can be solved numerically by solving back from the final period. This maximization is subject to the constraints in equation (10). I use standard methods of discretizing the wealth space to closely approximate the solution.

To calculate α, the Annuity Equivalent Wealth, one must first find the maximum utility V^* for the case in which the individual has the ability to fully annuitize W^*. Because this individual fully annuitizes, he starts off with zero non-annuitized wealth, $W_0 = 0$. One then solves for the case in which annuities are not available. That is, A_t is constrained to be zero for all t. It is then possible to solve for the amount of additional wealth, ΔW, which must be given to the individual in the absence of annuities such that the utility without annuities is equal to V^*.

That is, ΔW is defined such that:

$$V(W^* + \Delta W \mid A_t = 0, \forall t) = V^* \tag{15}$$

Annuity equivalent wealth is then defined as:

$$a = \frac{W^* + \Delta W}{W^*} \tag{16}$$

5. <u>Results with No Bequest Motives</u>

The Annuity Equivalent Wealth is calculated for individuals retiring at age 67, which is the Normal Retirement Age that the existing U.S. OASI system is transitioning towards. The cohort chosen for this study is that which enters the workforce at age 22 in the year 2000, since this is a cohort that has been used in several other studies of Social Security reform (Feldstein & Ranguelova 2000, Brown 2000). Results are quite similar for other cohorts. Within this cohort, we consider the mortality differentials across the gender, racial, ethnic and education groups described in section 2. While it is true that individuals in these demographic groups may enter retirement with substantially different levels of wealth, the CRRA utility function used in the simulations is invariant to the scale of wealth and therefore the annuity equivalent wealth measure, which is stated as a percentage of initial wealth, is unaffected by the differences in wealth levels across groups.

The value of annuitization is, however, related to the degree of risk aversion. In particular, more risk averse individuals will value annuities more highly than less risk averse individuals. While there is some evidence that risk aversion differs across segments of the population (Eisenhower & Halek 1999, Barsky, et al 1997), it is difficult to pin down these differences in a precise manner. Therefore, annuity equivalent wealth values are reported for all demographic groups for CRRA coefficients one through five. A risk aversion of one corresponds to log utility, a value that is often found to be the average risk aversion in many studies of consumption (Laibson, Repetto, & Tobacman 1998). Higher levels of risk aversion have been found in other studies, particularly those examining the equity premium puzzle, and thus annuity equivalent wealth results are reported for higher levels as well.

Table 3 reports the annuity equivalent wealth for the case of a constant real annuity that is uniformly priced for all individuals. There are several aspects of these numbers that are worth noting. First, as has been found in previous studies focusing on representative individuals, the utility gains from annuitization are quite high. Focusing on average men, the annuity equivalent wealth ranges from 1.35 at log utility to 1.546 for a risk aversion coefficient of 5. Second, even poorly educated black men, those with the worst mortality prospects of all the groups represented, have an annuity equivalent wealth of 1.296 when evaluated using log utility. Thus, even though the money's worth calculation indicates that poorly educated black men receive negative transfers on the order of –20% from being required to annuitize at a uniform price, the utility gains are still substantial. Third, as expected, annuity valuation is rising with risk aversion for all individuals.

Fourth, there is a surprising lack of significant dispersion in the annuity equivalent wealth figures across demographic groups. The largest effects are between men and women. With log utility, the difference between the utility gain to average women and that average men is 11.5% of wealth. This should be contrasted with the 15.6% difference when evaluated on purely a financial basis in Table 2. This difference shrinks to only 4.4% of wealth at risk aversion of 5. Within genders, there is very little difference. For example, the difference in annuity equivalent wealth of college educated white men and less than high school educated black men is only 6.5% of wealth when risk aversion is 1, and only 1.6% of wealth when risk aversion is 5. Again, this stands in stark contrast to the results when reported on purely a financial basis, where the difference in money's worth between these two groups was 16.7%.

These results may seem somewhat surprising given that the financial transfers are so large. It is important to realize, however, that much of the utility value of annuitization comes from the fact that it eliminates the risk of running resources down to a very low level in the event that one lives older than expected. While high mortality risk groups have a lower probability of this happening, that probability is still non-zero, and the utility gains from avoiding states of low consumption are quite large.

As an interesting comparison, annuity equivalent wealth results are next computed for the case in which annuities are priced for each demographic group on an actuarially fair basis. In other words, the annuity is "risk-class" priced, so groups with lower mortality rates receive lower annuity payments. From a financial perspective, the money's worth for every group is equal to one. Table 4 reports the difference in annuity payouts that arise under this pricing assumption. Note that the monthly payment ranges from a low of $553.08 for college educated white women, to a high of $776.92 for black men with less than a high school education.

Table 5 indicates that high mortality risk individuals value actuarially fair annuities far more highly than low mortality risk individuals. For example, with a risk aversion coefficient of one, a black male with less than a high school education has an Annuity Equivalent Wealth of 1.632, meaning that gaining access to actuarially fair annuity markets is equivalent to a 63.2% increase in non-annuitized wealth. This represents a doubling of the 32.1% increase in wealth for a college educated white female, despite the fact that the payments are only 40% higher. Once again, at higher levels of risk aversion, the annuity equivalent wealth rises for all demographic groups. At a risk aversion coefficient of 5, for example, the annuity equivalent wealth ranges from a low of 1.435 to a high of 1.929.

Results thus far suggest several interesting conclusions. First, even in an environment in which annuities are uniformly priced, if administrative costs are zero, all consumers are made better off by availability of the annuity. Second, while the degree of redistribution appears large when measured on a financial basis, the degree of redistribution when measured on a utility-adjusted basis is substantially smaller. Third, if annuities are not uniformly priced, but rather are priced based on the mortality experience of each risk class, the high mortality risk groups benefit the most from annuitization.

High mortality risk groups experience a low money's worth due to the fact that they are less likely to be alive in future periods to consume the annuity payments. As such, as was shown in table 2, they are better off from a financial perspective if the annuity is declining in real terms. This is because a declining real annuity front-loads payments into early periods, when the individual is more likely to be alive. As was also demonstrated in section 2, if an individual's mortality rate is higher than that used in the pricing of annuities, he will prefer a consumption path that with a downward tilt. Fixing annuities in nominal terms and letting inflation erode its real value over time is an example of a product that would provide such a downward slope. Table 6 reports annuity equivalent wealth results for the case of an annuity that declines at a real rate of 3% per annum. This rate is roughly consistent with the average historical rate of inflation in the

U.S. Comparing the results from table 6 with those of table 3 (constant real annuities), it is clear that most individuals are made worse off by having the annuity decline in real terms. In fact, for risk aversion over 2, every individual is better off with constant real annuities. Only in the case of log utility is any group made better off by declining annuities, and one might expect, these are high mortality risk groups. Specifically, whites with less than a high school education, and black men of all education levels, are the only groups to do better under a declining real annuity with log utility. While the dispersion in annuity equivalent wealth does decrease, thus decreasing the amount of redistribution, it does so mainly by depressing the value of annuities for most groups, rather than raising it for many. It should also be noted that these are results for a declining real annuity. A true nominal annuity that is subject to inflation risk, would lower annuity values for everyone. Thus, from a utility standpoint, it seems that front-loading payments through imperfect inflation-indexing is not a satisfactory way to handle distributional concerns.

All of the above results assume that annuities do not have any additional costs, i.e., that they are actuarially fair for the average annuitant. However, it is unlikely that annuities can be provided with no administrative costs. For example, private annuity markets in the U.S. are estimated to have administrative costs of approximately 8% (Mitchell, et al 1999). Table 7 shows annuity equivalent wealth results for the case of a uniform price, constant real annuity with 8% administrative costs. Not surprisingly, all the annuity valuations fell relative to table 3 by approximately 8%.

6. Bequest Motives and Bequest Options

These results, like those in most of the annuity valuation literature, have been limited to life-cycle consumers who do not value bequests. While this may be due in part to analytical convenience, it is in much larger part due to the lack of consensus in the literature on the importance of bequests and how to model them. A debate in the literature during the 1980s between Kotlikoff & Summers (1981) and Modigliani (1988) suggested that the fraction of the aggregate US capital stock that can be accounted for by intergenerational transfers ranges from

20% to nearly 80%. Simulation models (e.g., Laitner 1990, Lord and Rangazas 1991) have also generated a wide range of estimates. Direct surveys provide a third source of evidence, and suggest that bequests may account for up to 30% of wealth (Aaron and Munnell 1992). Attempts to measure the strength of bequest motives through an examination of wealth decumulation patterns in retirement (Hurd 1987, 1989), life insurance holdings (Bernheim 1991, Brown 1999) and annuity decisions in defined contribution plans (Brown 2001) have also lead to conflicting results.

There is little more agreement on the proper way to model bequest motives. There are at least four classes of model, explained in greater detail by Gale & Slemrod (2000). These include the accidental bequest motive (Abel 1985), pure altruism (Barro 1974), exchange models (Bernheim, Shleifer & Summers 1985, Cox 1987), and models in which individuals obtain utility from the wealth left as a bequest (Carroll 2000, Aaron & Munnell 1992). As noted by Gale and Slemrod (2000), "each motive is plausible and draws support from at least some research, but each motive that has been tested has also been rejected."

Given this background, a complete exploration of alternative bequest models is well beyond the scope of this paper. Furthermore, for purposes of valuing annuities, it is not clear that much will be learned from such an examination. Most specifications, with the exception of the accidental bequest motive, are going to have a similar implication that complete annuitization is no longer optimal because individuals will wish to keep some wealth in a bequeathable form. To illustrate this broader point, this paper chooses one particular specification of bequests, and models them as entering additively into the utility function as follows:

$$Max_{\{C_t\}} \sum_{t=1}^{T-age+1} \frac{P_t U(C_t) + q_t P_{t-1} B(W_t)}{(1+r)^t} \tag{17}$$

where $B(W_t)$ is the utility of bequests function defined over the amount of wealth available in period t for bequeathing. The probability of leaving a bequest in each period is equal to the probability of dying in period t, conditional on surviving to period t-1. This setup assumes that bequest utility is discounted using the same rate of time preference as exhibited for consumption, though this need not be the case (see Jousten 1998). $B(W_t)$ is modeled as a constant relative risk aversion utility function and that the risk aversion parameter is the same as for consumption. β is a weighting factor that can be varied to change the relative weight of consumption versus bequests.

The dynamic programming algorithm can be adapted to include a bequest motive. However, there are several conceptual difficulties that must be considered in thinking about the relevant calculations to do. In considering the payout phase of a retirement system, it matters to what extent individuals have wealth outside of the pension. For example, if policymakers are designing the payout rules for an individual accounts system, full annuitization of account balances may be optimal if the individual has substantial non-annuitized wealth in other accounts. As long as the individual's desired consumption path is greater than or equal to the amount of the annuity payment financed by the individual account, full annuitization of the account remains optimal. On the other hand, if for some reason individuals fail to adequately save outside of the pension system and therefore wish to leave part of their pension account as a bequest, then complete annuitization is no longer desired.

If people are over-annuitized by a mandatory annuitization system and wish to convert some of the annuity wealth into a bequeathable asset, one way to do this is through the purchase of a life insurance contract (Bernheim 1991). While recent evidence suggests that very few elderly individuals purchase insurance for this reason (Brown 1999) in today's pension environment, this option would be available if mandatory annuitization increased further. If people can save adequately outside of the annuitized pension and/or have access to life insurance markets, there is not much to be gained from running additional simulations of annuity value with a bequest motive incorporated. In some cases, however, individuals may not save outside the pension plan, or may be unable to qualify for life insurance with which to offset the annuity. We can examine this special case by incorporating bequest motives into the annuity equivalent wealth calculation. There are three questions that are of interest in this situation. First, in the presence of bequest motives, would complete annuitization still be welfare-improving? Second, how much better off would consumers be if they were permitted to partially annuitize their accounts, leaving some portion for bequests? Third, are annuities that offer bequest options, such as period certain guarantees, a useful tool for handling bequest motives and reducing redistribution across groups?

Table 8 reports annuity equivalent wealth results for complete annuitization by individuals with bequest motives. In each case, $B(W_t)$ is modeled as a CRRA utility function with the same risk aversion parameter as the U(.). Results are reported for risk aversion coefficients of 1 and 5, and the final column indicates what happens to the annuity equivalent wealth when risk aversion rises to 3 or greater. In addition, results are reported for two different values (0.5 and 1.0) of β, the weighting factor on bequest utility. In order to avoid the infinite disutility that occurs when a person leaves a zero bequest, the simulations of "complete" annuitization allows them to retain 0.01% of wealth as a bequest. For a person with $100,000 in retirement wealth, this is like saying they died with a $10 bill in their pocket, which is left as a bequest. Note also that as time goes on, the individual can increase the size of the bequest by saving a portion of each annuity payment.

The results indicate that with a bequest rating parameter of β=0.5 and a risk aversion coefficient of 1, the annuity equivalent wealth for an average male is 1.233, significantly lower than the annuity equivalent wealth in the absence of a bequest motive. The annuity equivalent wealth for an average female is 1.373, also lower than in the absence of bequests. Increasing β to one we see that all log utility consumers still value annuities, even those with very high mortality rates.

As risk aversion increases to 2, however, the annuity equivalent wealth falls below one for all men and all but the lowest mortality risk women. With risk aversion of 2 and β of 1, everyone has an annuity equivalent wealth less than one. Importantly, and annuity equivalent wealth of less than one indicates that the annuity has actually made the individual worse off than without. An annuity equivalent wealth of 0.7 can be interpreted as saying that complete mandatory annuitization is equivalent in utility terms to a 30% tax on wealth. When risk aversion reaches 3 and above, the annuity equivalent wealth value crashes to less than 0.2, in some cases approaching 0, indicating that from a utility perspective, mandatory complete annuitization is like a confiscatory tax on individuals with bequest motives.

Keep in mind, however, that these results are very much a lower bound on the value of utility for these individuals, because it assumes that the person has no other wealth with which to leave a bequest, and cannot undo the annuity through the purchase of life insurance. If, instead, we assume that partial annuitization is permitted, we can then ask what fraction of wealth the consumer would annuitize, and how much the partial annuitization is worth. To do this complicates the dynamic programming procedure by requiring that one first find the optimal amount of annuitization, and then compute the annuity equivalent wealth for that amount. This is implemented using a grid search of the utility achieved over the fraction of wealth annuitized. To reduce the computational burden, it is assumed that the individual will annuitize an integer percentage of wealth.

Table 9 reports these results, showing both the fraction of wealth optimally annuitized (to the nearest 1%) and the annuity equivalent wealth associated with that level of wealth, for the case of β=1. Note that when partial annuitization is permitted, the denominator of the annuity equivalent wealth calculation is the amount of *annuitized* wealth. If one is interested in the gain in wealth relative to total wealth, one can simply transform the measure by taking 1+(annuity equivalent wealth-1)*(% annuitized). Not surprisingly, the individuals with the highest mortality risk wish to annuitize the smallest fraction of wealth. For example, black men with less than a high school education and log utility would prefer to annuitize only 69% of their wealth, and doing so gives them an annuity equivalent wealth of 1.138. At higher levels of risk aversion, the fraction annuitized rises for men (who have mortality rates below those used in pricing) and falls for women (who have rates higher than those used in pricing). By the time risk aversion reaches a level of 5, the fraction annuitized appears to congregate around 91-93% for everyone, with annuity valuations ranging from 1.462 to 1.530. It is quite clear from these results that optimal policy should permit individuals with bequest motives to partially annuitize, and leave part of their wealth in bequeathable form.

A third natural question is whether or not various "bequest options" are a valuable add-on to a retirement annuity. In particular, is it desirable to allow individuals to purchase "period certain" options that provide annuitants with a guaranteed minimum number of payments, regardless of the survival of the insured individual? For example, a "life with 10 year certain" annuity will make 10 years of guaranteed payments, and then will continue to pay off in additional months if and only if the insured individual is still alive. These products are extremely popular in the U.S. (Feldstein & Ranguelova 2000, Brown & Poterba 2000).

As a bequest option, period certain products are somewhat unusual in that the bequest that they provide is declining in value over time and is worthless at the end of the guarantee period. Naturally, individuals can save a portion of their annuity payment each period in order to provide for a bequest later in life. But this raises the natural question of why such an individual would not simply prefer to leave some portion of wealth in bequeathable form and then annuitize the rest in a straight life annuity.

In fact, it is not necessary to resort to complex utility-based calculations to demonstrate that a life annuity with a period certain option is dominated by allowing partial annuitization. To understand why, it is useful to consider the pricing equation for a life annuity with an X-period certain provision, where A_{PC} is the annuity payment for a period certain annuity.

$$W = \sum_{t=1}^{X} \frac{A_{PC}}{(1+r)^t (1+p)^t} + \sum_{t=X+1}^{T-age+1} \frac{A_{PC} P_t}{(1+r)^t (1+p)^t} \tag{18}$$

The first summation is simply the annuity payment for the period of the guarantee, discounted to time 0. Because the annuity pays off during the guarantee period regardless of whether the annuitant survives or not, no mortality rates enter the calculation. The second summation is the pure life annuity component, which does incorporate mortality rates because it only pays off in periods t>X if the individual is still alive. In fact, as suggested by Doyle, Mitchell & Piggott (2001), this product can be viewed as a combination of two products. The first is a non-life contingent product, and the second is a life annuity that is deferred for X periods.

If we then decompose the equation for a straight life annuity (denoted A_L) into two parts, its present value can be written as:

$$W^* = \sum_{t=1}^{X} \frac{A_L P_t}{(1+r)^t (1+p)^t} + \sum_{t=X+1}^{T-age+1} \frac{A_L P_t}{(1+r)^t (1+p)^t} \tag{19}$$

The only difference between the two equations is the presence or absence of survival probabilities in the summation over the first X periods. For a common W, $A_{PC}<A_L$. Now assume that the individual fully annuitizes in the period certain annuity and find the value of A_{PC}. Through simple algebraic manipulation, one can find the portion of wealth, ψ, that can be left un-annuitized such that when one annuitizes $(1-\psi)W$ in a straight life annuity, $A_L=A_{PC}$. This un-annuitized portion, ψ, is simply equal to the present expected value of payments made in the first X periods, where expectations are taken with respect to those states in which the insured individual is deceased, as follows:

$$y = \sum_{t=1}^{X} \frac{1-P_t}{(1+r)^t (1+p)^t} \tag{20}$$

Therefore, if one leaves fraction ψ of wealth un-annuitized, the monthly annuity payment available to the annuitant is the same as if he had purchased an X-period certain annuity. Unlike the payments from an X-period certain annuity, however, these payments cease when the individual dies. However, the amount of money that is guaranteed as a bequest, ψ, is equivalent to the expected value of the bequest that would have been left via a period certain contract.

Therefore, the consumption profile available from these two approaches is identical, and the expected value of the bequests is the same. The key difference, however, is that under the partial annuitization approach, ψ is known with certainty and available at time 0. With the period certain approach, the size of the bequest left from the period certain contract is uncertain and time varying. Therefore, if the insured individual is risk averse with respect to the size of bequests, he will prefer partial annuitization over a period certain contract. If he is risk neutral over bequests (i.e., if bequests enter utility linearly), he will be indifferent between the two approaches. Therefore, unless the insured individual has highly unusual preferences over bequests, partial annuitization should always preferred to annuitization with a period certain option. It should also be noted that the money's worth measure of redistribution is identical under these two approaches. This is because the present value of the bequeathable portion of wealth in the two scenarios is identical.

Therefore, to the extent that individuals behave according to life cycle theory but have bequest motives, it would appear that permitting partial lump-sum distributions is preferred to any alternative strategy. However, there may be other behavioral reasons why individuals overwhelmingly choose period certain options, even when annuitization is not mandatory. For example, people may experience a form of "*ex ante* regret," i.e., they may incur psychological costs from worrying about the fact that if they die soon after purchasing a life annuity, they will have made, on an *ex-post* basis, a poor investment. While this should be true of partial annuitization as well, casual empiricism suggests that people feel emotionally better about knowing that their heirs will get back at least part of their original investment if they die early. Nonetheless, in a strictly rational model of life-cycle consumers with bequest motives, period certain options are inferior to partial annuitization. Designers of a pension payout system that wish to reduce the degree of financial redistribution can achieve the same distributional goals through partial annuitization, while increasing the utility of the individuals.

7. Conclusions

Annuities provide valuable longevity insurance to individual with uncertain lifetimes. However, mandating that all individuals annuitize at a uniform price also has distributional implications. When measured on a financial basis, these transfers can be quite large and often away from economically disadvantaged groups and towards groups that are better off financially. This paper indicates, however, that the insurance value of annuitization is sufficiently large that, relative to a world with no annuities, all groups can be made better off through a mandatory annuitization system, so long as administrative costs are kept low. In particular, even groups with mortality rates far higher than those used to price the annuities are made better off than in the absence of annuities. Furthermore, there appears to be far less redistribution when evaluated on a utility adjusted basis.

When the model is extended to include bequest motives, full annuitization of pension plans may still be optimal if individuals can do enough saving outside of the pension system to satisfy their bequest needs, or if they have access to life insurance markets. However, in those cases where an individual is over-annuitized for bequest reasons, and does not have access to other resources, complete annuitization is inferior to partial annuitization. Results also indicate that adding various bequest options to life annuities, such as period certain options, is inferior to allowing partial annuitization.

These results are based on a counterfactual world in which no annuities are available. This is not as extreme a counterfactual as it may at first appear, given that outside of Social Security and some defined benefit plans, annuity markets in the U.S. are quite thin (Brown et al 2001). It is important to note, however, that the distributional consequences of mandating annuitization in an individual accounts system, for example, will differ depending on the counterfactual. If, for example, individuals are already fully annuitized in a system that prices annuities uniformly, then moving to a system of individual accounts that does the same thing will not have any new distributional effects (aside from differences in the accumulation phase). If individuals are already annuitized in a system that prices annuities separately for each demographic group, then the move to a uniform priced annuity system would clearly represent a shift in resources away from high risk individuals to low risk individuals. The story becomes even more complex if, in the counterfactual world, annuities are available on a voluntary basis only, and at high cost (a fair representation of the individual annuity market in the U.S.) Mandating annuitization can raise the average payout rate by forcing individuals into the market, improving the welfare of those who had been annuitizing previously. It would also improve the welfare of those that would have liked to annuitize but did not due to the cost structure.

It should also be noted that a full social welfare comparison of alternative annuity systems would require the specification of an explicit social welfare function. Recent work by Sheshinski (1999) demonstrates conditions under which uniform annuity pricing can in fact be social welfare maximizing, and conditions under which it is not.

This paper has focused exclusively on longevity insurance. One potentially fruitful area for future research would be an investigation of the utility value of other insurance aspects of public pension systems, such as disability, survivor and dependent benefits. To the extent that these programs have substantial insurance value, previous studies of the distributional effects of Social Security that have ignored this value may not tell the complete story about the distributional effects of social insurance programs.

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TABLE 1
Conditional Life Expectancy by Gender, Race, Hispanic Status, and Education

	Conditional Life Expectancy at age 22		Conditional Life Expectancy at age 67	
	Men	Women	Men	Women
All	77.4	83.4	83.5	87.2
All Whites	78.3	84.0	83.6	87.4
All Blacks	71.8	80.0	82.3	86.1
All Hispanics	77.4	85.2	83.5	88.3
Whites: College +	80.5	85.1	84.4	87.8
Whites: HS +	77.8	83.9	83.4	87.3
Whites: < HS	75.3	82.1	82.3	86.5
Blacks: College +	75.7	81.9	83.4	86.8
Blacks: HS +	71.6	80.0	82.2	86.1
Blacks: < HS	68.1	77.5	81.0	85.1

Notes: "Conditional Life Expectancy" is used to describe the age to which an individual can expect to live, conditional on attaining age 22 or 67.

Source: Author's calculations as described in text.

TABLE 2
Money's Worth of Annuities

	Real Annuity $r = .03$	Nominal Annuity $r = \pi = .03$	Real Annuity with 20-year Period Certain
MEN			
All	0.920	0.938	0.972
All Whites	0.927	0.944	0.973
All Blacks	0.862	0.886	0.964
All Hispanics	0.920	0.938	0.972
Whites: College +	0.967	0.980	0.978
Whites: HS +	0.916	0.934	0.973
Whites: < HS	0.865	0.889	0.964
Blacks: College +	0.916	0.935	0.970
Blacks: HS +	0.857	0.881	0.964
Blacks: < HS	0.800	0.830	0.955
WOMEN			
All	1.076	1.059	1.026
All Whites	1.084	1.067	1.027
All Blacks	1.022	1.011	1.018
All Hispanics	1.123	1.097	1.042
Whites: College +	1.106	1.086	1.030
Whites: HS +	1.080	1.063	1.027
Whites: < HS	1.044	1.031	1.022
Blacks: College +	1.055	1.041	1.023
Blacks: HS +	1.022	1.011	1.018
Blacks: < HS	0.976	0.970	1.011

Source: Author's calculations as described in text

TABLE 3
Annuity Equivalent Wealth Under Uniform Pricing

	CRRA=1	CRRA=2	CRRA=3	CRRA=4	CRRA=5
MEN					
All	1.350	1.449	1.497	1.527	1.546
All Whites	1.352	1.450	1.498	1.528	1.546
All Blacks	1.328	1.437	1.488	1.522	1.542
All Hispanics	1.350	1.449	1.497	1.527	1.546
Whites: College +	1.361	1.452	1.498	1.527	1.546
Whites: HS +	1.351	1.451	1.499	1.529	1.548
Whites: < HS	1.325	1.434	1.486	1.520	1.540
Blacks: College +	1.343	1.443	1.492	1.523	1.542
Blacks: HS +	1.328	1.437	1.488	1.523	1.543
Blacks: < HS	1.296	1.415	1.472	1.511	1.534
WOMEN					
All	1.465	1.531	1.560	1.577	1.588
All Whites	1.465	1.531	1.560	1.577	1.588
All Blacks	1.459	1.529	1.560	1.577	1.588
All Hispanics	1.487	1.545	1.570	1.585	1.597
Whites: College +	1.466	1.530	1.559	1.576	1.588
Whites: HS +	1.465	1.531	1.561	1.577	1.588
Whites: < HS	4.463	1.531	1.562	1.578	1.589
Blacks: College +	1.462	1.530	1.560	1.577	1.588
Blacks: HS +	1.459	1.529	1.561	1.577	1.588
Blacks: < HS	1.453	1.526	1.560	1.577	1.587

Source: Author's calculations as described in text

BC1.g

TABLE 4
Monthly Income from $100,000 Policy if Priced Based on Group Specific Mortality

	Monthly Income	
	Men	Women
All	$675.36	$577.36
All Whites	670.42	572.90
All Blacks	720.83	608.15
All Hispanics	675.36	553.08
Whites: College +	642.73	561.83
Whites: HS +	678.25	575.13
Whites: < HS	718.40	595.19
Blacks: College +	678.22	589.01
Blacks: HS +	725.13	608.01
Blacks: < HS	776.92	636.84

Source: Author's calculations as described in text

TABLE 5
Annuity Equivalent Wealth with Actuarially Fair Risk Class Pricing

	CRRA=1	CRRA=2	CRRA=3	CRRA=4	CRRA=5
MEN					
All	1.471	1.578	1.633	1.665	1.688
All Whites	1.462	1.568	1.622	1.653	1.675
All Blacks	1.548	1.675	1.737	1.774	1.799
All Hispanics	1.471	1.578	1.633	1.665	1.688
Whites: College +	1.409	1.504	1.553	1.582	1.601
Whites: HS +	1.479	1.587	1.643	1.674	1.697
Whites: < HS	1.539	1.666	1.728	1.766	1.791
Blacks: College +	1.470	1.578	1.635	1.668	1.691
Blacks: HS +	1.557	1.686	1.748	1.786	1.810
Blacks: < HS	1.632	1.783	1.859	1.900	1.929
WOMEN					
All	1.359	1.421	1.499	1.465	1.476
All Whites	1.349	1.410	1.437	1.454	1.464
All Blacks	1.427	1.496	1.527	1.543	1.553
All Hispanics	1.318	1.372	1.396	1.410	1.419
Whites: College +	1.321	1.380	1.408	1.424	1.435
Whites: HS +	1.354	1.416	1.443	1.460	1.470
Whites: < HS	1.399	1.466	1.495	1.512	1.521
Blacks: College +	1.384	1.449	1.478	1.495	1.505
Blacks: HS +	1.426	1.496	1.526	1.543	1.553
Blacks: < HS	1.489	1.565	1.599	1.615	1.629

Source: Author's calculations as described in text

BC2.g

TABLE 6
Annuity Equivalent Wealth Under Uniform Pricing
Nominal (Declining Real) Annuity

	CRRA=1	CRRA=2	CRRA=3	CRRA=4	CRRA=5
MEN					
All	1.350	1.419	1.441	1.446	1.447
All Whites	1.351	1.419	1.440	1.446	1.447
All Blacks	1.339	1.419	1.441	1.447	1.446
All Hispanics	1.350	1.419	1.441	1.446	1.447
Whites: College +	1.354	1.416	1.438	1.445	1.449
Whites: HS +	1.351	1.421	1.442	1.447	1.447
Whites: < HS	1.338	1.418	1.441	1.447	1.446
Blacks: College +	1.346	1.416	1.439	1.446	1.447
Blacks: HS +	1.340	1.420	1.442	1.447	1.446
Blacks: < HS	1.321	1.414	1.439	1.449	1.445
WOMEN					
All	1.408	1.441	1.447	1.448	1.450
All Whites	1.408	1.441	1.447	1.448	1.450
All Blacks	1.409	1.444	1.477	1.448	1.451
All Hispanics	1.417	1.443	1.448	1.448	1.450
Whites: College +	1.406	1.439	1.477	1.449	1.449
Whites: HS +	1.408	1.441	1.447	1.448	1.450
Whites: < HS	1.410	1.443	1.447	1.448	1.451
Blacks: College +	1.408	1.442	1.447	1.448	1.451
Blacks: HS +	1.409	1.444	1.447	1.448	1.451
Blacks: < HS	1.409	1.445	1.446	1.448	1.451

Source: Author's calculations as described in text

BC3.g

TABLE 7
Annuity Equivalent Wealth Under Uniform Pricing with 8% Load Factor

	CRRA=1	CRRA=2	CRRA=3	CRRA=4	CRRA=5
MEN					
All	1.243	1.332	1.380	1.407	1.426
All Whites	1.245	1.333	1.381	1.408	1.427
All Blacks	1.223	1.320	1.373	1.402	1.421
All Hispanics	1.243	1.332	1.380	1.407	1.426
Whites: College +	1.252	1.335	1.381	1.408	1.428
Whites: HS +	1.244	1.334	1.382	1.409	1.428
Whites: < HS	1.221	1.318	1.371	1.400	1.419
Blacks: College +	1.237	1.326	1.375	1.403	1.423
Blacks: HS +	1.223	1.321	1.374	1.403	1.421
Blacks: < HS	1.194	1.303	1.359	1.392	1.411
WOMEN					
All	1.349	1.410	1.438	1.455	1.465
All Whites	1.349	1.410	1.438	1.455	1.465
All Blacks	1.343	1.410	1.438	1.455	1.466
All Hispanics	1.368	1.423	1.448	1.462	1.470
Whites: College +	1.349	1.409	1.437	1.453	1.464
Whites: HS +	1.349	1.411	1.438	1.455	1.466
Whites: < HS	1.347	1.412	1.439	1.456	1.467
Blacks: College +	1.346	1.410	1.438	1.455	1.466
Blacks: HS +	1.343	1.410	1.438	1.455	1.466
Blacks: < HS	1.337	1.408	1.437	1.455	1.465

Source: Author's calculations as described in text

BC4.g

TABLE 8
Annuity Equivalent Wealth
Complete* Mandatory Annuitization with Bequest Motives

	Life Annuity CRRA=1		Life Annuity CRRA=2		Life Annuity CRRA ≥ 3
	β=0.5	β=1.0	β=0.5	β=1.0	β∈{0.5,1.0}
MEN					
All	1.233	1.166	0.853	0.623	< 0.2
All Whites	1.237	1.171	0.871	0.641	< 0.2
All Blacks	1.198	1.121	0.726	0.501	< 0.2
All Hispanics	1.233	1.166	0.853	0.623	< 0.2
Whites: College +	1.255	1.196	0.972	0.749	< 0.2
Whites: HS +	1.233	1.165	0.846	0.614	< 0.2
Whites: < HS	1.197	1.121	0.742	0.515	< 0.2
Blacks: College +	1.226	1.159	0.850	0.621	< 0.2
Blacks: HS +	1.196	1.118	0.716	0.493	< 0.2
Blacks: < HS	1.151	1.065	0.605	0.405	< 0.2
WOMEN					
All	1.373	1.317	1.094	0.870	< 0.2
All Whites	1.375	1.321	1.116	0.895	< 0.2
All Blacks	1.354	1.289	0.960	0.721	< 0.2
All Hispanics	1.402	1.352	1.178	0.968	< 0.2
Whites: College +	1.380	1.329	1.167	0.961	< 0.2
Whites: HS +	1.374	1.319	1.107	0.884	< 0.2
Whites: < HS	1.363	1.302	1.019	0.783	< 0.2
Blacks: College +	1.365	1.306	1.046	0.814	< 0.2
Blacks: HS +	1.354	1.290	0.962	0.723	< 0.2
Blacks: < HS	1.335	1.264	0.859	0.608	< 0.2

Source: Author's calculations as described in text
* "Complete" annuitization means that 99.99% of wealth is annuitized. 0.01% is assumed to left un-annuitized so as to prevent infinite disutility from the inability to leave a bequest.

TABLE 9
Annuity Equivalent Wealth
Partial Annuitization with Bequest Motives (b=1)

	Life Annuity CRRA=1		Life Annuity CRRA=5	
	% Ann.	Annuity equivalent wealth	% Ann.	Annuity equivalent wealth
MEN				
All	83%	1.193	93	1.480
All Whites	84	1.196	93	1.481
All Blacks	78	1.167	92	1.473
All Hispanics	83	1.193	93	1.480
Whites: College +	92	1.212	93	1.482
Whites: HS +	83	1.194	93	1.482
Whites: < HS	78	1.166	92	1.472
Blacks: College +	83	1.188	93	1.476
Blacks: HS +	77	1.166	92	1.474
Blacks: < HS	69	1.138	91	1.462
WOMEN				
All	97	1.323	93	1.529
All Whites	98	1.326	93	1.529
All Blacks	93	1.305	93	1.529
All Hispanics	99	1.323	93	1.536
Whites: College +	98	1.333	93	1.527
Whites: HS +	97	1.325	93	1.529
Whites: < HS	93	1.314	93	1.530
Blacks: College +	93	1.315	93	1.529
Blacks: HS +	93	1.305	93	1.529
Blacks: < HS	93	1.287	92	1.527

Source: Author’s calculations as described in text