# Interdependent Durations in Joint Retirement

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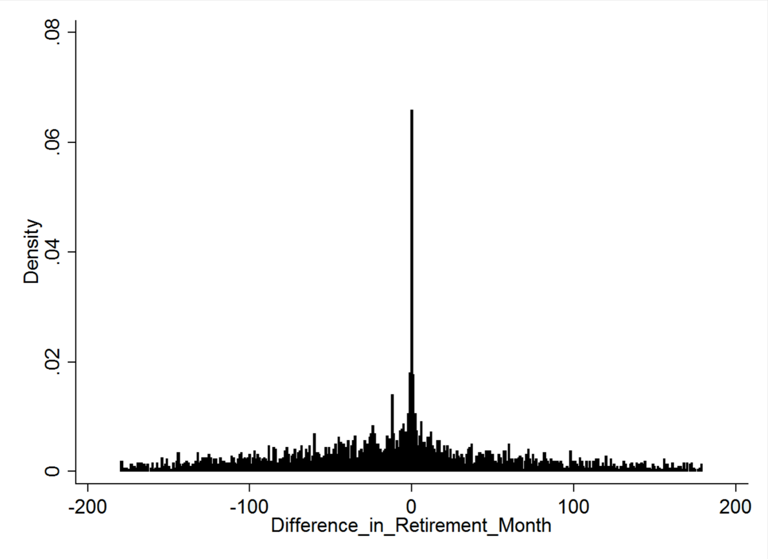
# Motivation


Density
0
.02
.04
.06
.08
-200
-100
0
100
200
Difference_in_Retirement_Month

# Combining AFT and MPH

Mixed Proportional Hazard Model

$$\ln(Z(T)) = -\ln(\varphi(x)) - \ln(\nu) + \eta \qquad \text{(MPH)}$$

where $\eta \sim \ln(-\ln(U(0,1)))$.

Accelerated Failure Time Model

$$\log T = x\beta + \log T^* \qquad \text{(AFT)}$$

where the distribution of $\log T^*$ is unspecified.

*MPH* $\cup$ *AFT* = *GAFT*

$$\ln(Z(T)) = -\ln(\varphi(x)) + \varepsilon \qquad \text{(GAFT)}$$

where the distribution of $\varepsilon$ is unspecified.

# We want to think about simultaneous durations.

- We want to introduce dependence of durations in a "structural" way and not only through unobservables.
- First review what we do in linear regressions

# Seemingly Unrelated Regression

$$
\begin{aligned}
y_1 &= x_1'\beta_1 + \varepsilon_1 \\
y_2 &= x_2'\beta_2 + \varepsilon_2
\end{aligned}
$$

(not what we want to generalize)

# Triangular Systems

$$
\begin{aligned}
y_1 &= x_1'\alpha_1 + \varepsilon_1 \\
y_2 &= y_1\gamma_2 + x_2'\alpha_2 + \varepsilon_2
\end{aligned}
$$

(also not quite what we want to generalize)

# Simultaneous Equations

$$
\begin{aligned}
y_1 &= y_2\gamma_1 + x_1'\alpha_1 + \varepsilon_1 \\
y_2 &= y_1\gamma_2 + x_2'\alpha_2 + \varepsilon_2
\end{aligned}
$$

(what we want to generalize!)

# Statistical approach

We could simply specify

$$p_{T_1|T_2=t_2}(t) = \begin{cases} \pi_1(t_2) & \text{if } t = t_2 \\ f_1(t)(1-\pi_1(t_2)) & \text{otherwise.} \end{cases}$$

$$p_{T_2|T_1=t_1}(t) = \begin{cases} \pi_2(t_1) & \text{if } t = t_1 \\ f_2(t)(1-\pi_2(t_1)) & \text{otherwise.} \end{cases}$$

(functional form not essential)

Why reasonable from an economic point of view?

# Our approach

We will think of $T_1$ and $T_2$ as chosen by individuals.

We will allow for models where $T_1$ and $T_2$ are each continuous, but $P(T_1 = T_2) > 0$.

We want the effect to not only be through the hazard (although that is often the most reasonable).

# Our approach

- Honoré and de Paula [2010]: durations are Nash Equilibria of a game theoretic model.
- Game theoretic model clearly not suitable when agents can coordinate but some of the features seem right.
- So we replace Nash Equilibrium with Nash Bargaining.

# Nash Bargaining (Zeuthen)

$$\max_{t_1,t_2} \left(u_1\left(t_1;t_2\right)-a_1\right)\left(u_2\left(t_2;t_1\right)-a_2\right)$$

where (for $i \neq j \in \{1,2\}$)

$$u_i(t_i;t_j) \equiv \int_0^{t_i} K_i e^{-\rho s} ds + \int_{t_i}^{\infty} Z(s)\varphi(x_i)\delta(s \geq t_j) e^{-\rho s} ds$$

Can be motivated aximatically

- Pareto Optimality.
- Independence of Irrelevant Alternatives.
- A Certain Symmetry.

# Simultaneous Equations GAFT

As in Honoré and de Paula [2010], this will lead to durations of the form

$$\ln\left(Z\left(T_i\right)\right) = -\ln\left(\varphi(x_i)\right) + \ln\left(K_i\right)$$

or the form

$$\ln\left(Z\left(T_i\right)\right) = -\ln\left(\varphi(x_i)\right) - \delta + \ln\left(K_i\right)$$

for some draws of $(K_1, K_2)$.

So this is a generalization of the GAFT.

# Implementation: Indirect Inference

Suppose that rather than doing MLE in the true model with parameter $\theta$, you do it in some approximate (*auxiliary*) model with parameter $\beta$, then

$$\widehat{\beta} = \arg\max_b \sum_{i=1}^{n} \log \mathcal{L}_a(b; z_i)$$
$$\xrightarrow{p} \arg\max_b E_{\theta_0}\left[\log \mathcal{L}_a(b; z_i)\right] \equiv \beta_0(\theta_0)$$

If we knew the right–hand–side as a function of $\theta_0$, then we could use this to solve the equation

$$\widehat{\beta} = \beta_0\left(\widehat{\theta}\right)$$

Of course, the problem is that we don't know

$$\beta_0(\theta) \equiv \arg\max_b E_\theta \left[\log \mathcal{L}_a\left(b; z_i\right)\right]$$

*But we can simulate it!!!*

# Auxiliary Models

- Weibull Proportional Hazard models for man and woman
  $\Rightarrow \mathcal{L}_{men}, \mathcal{L}_{women}$, timing of retirement

- Ordered Logit Model:
  $P(t_h > t_w|x), P(t_h = t_w|x), P(t_h < t_w|x)$
  $\Rightarrow \mathcal{Q}$, pervasiveness of joint retirement

- Overall auxiliary model pseudo-loglikelihood:
  $\ln \mathcal{L}_{men} + \ln \mathcal{L}_{women} + \ln \mathcal{Q}$

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- Weibull Proportional Hazard models for man and woman
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  $\Rightarrow \mathcal{Q}$, pervasiveness of joint retirement
- Overall auxiliary model pseudo-loglikelihood:
  $\ln \mathcal{L}_{men} + \ln \mathcal{L}_{women} + \ln \mathcal{Q}$

  Other auxiliary models?

# Retirement

We use data from the Health and Retirement Study.

If the respondent is not working and not looking and there is any mention of retirement through the employment status or the questions asking whether he/she considers him/herself retired, he/she is classified as retired.

# Retirement

Some important factors for retirement timing decision:

- Private pensions (especially DB);
- Health insurance;
- Savings (control using wealth variables);
- and... spouse decisions.
  Hurd (1989, 1990), Coile (1999, 2004a, b), Gustman and Steinmeier (2000, 2004), Blau (1997, 1998), Maestas (2001), Michaud (2003), Michaud and Vermeulen (2004), An, Jesper Christensen and Gupta (2004), Banks, Blundell and Casanova (2007), Casanova (2009)

We focus on retirement from the age of 60 (oldest in household) conditional on covariates at that point.

WIVES' Proportional Hazards (Weibull Baseline)

| **Variable** | **Coef.** (Std. Err.) | **Coef.** (Std. Err.) | **Coef.** (Std. Err.) | **Coef.** (Std. Err.) | **Coef.** (Std. Err.) | **Coef.** (Std. Err.) |
|---|---|---|---|---|---|---|
| $\alpha$ | 1.227 ** | 1.234 ** | 1.237 ** | 1.239 ** | 1.245 ** | 1.247 ** |
| | ( 0.042 ) | ( 0.043 ) | ( 0.043 ) | ( 0.044 ) | ( 0.045 ) | ( 0.045 ) |
| Constant | -5.840 ** | -5.978 ** | -5.792 ** | -6.003 ** | -5.943 ** | -5.986 ** |
| | ( 0.185 ) | ( 0.238 ) | ( 0.270 ) | ( 0.321 ) | ( 0.319 ) | ( 0.320 ) |
| Age Diff. | -0.068 ** | -0.068 ** | -0.067 ** | -0.070 ** | -0.070 ** | -0.070 ** |
| | ( 0.010 ) | ( 0.011 ) | ( 0.011 ) | ( 0.011 ) | ( 0.011 ) | ( 0.011 ) |
| V. G. Health | | | -0.200 | -0.237 | -0.285 † | -0.278 |
| | | | ( 0.152 ) | ( 0.167 ) | ( 0.167 ) | ( 0.169 ) |
| Good Health | | | -0.321 * | -0.384 * | -0.416 * | -0.409 |
| | | | ( 0.159 ) | ( 0.172 ) | ( 0.171 ) | ( 0.173 ) |
| Health Ins. | | | | -0.020 | -0.019 | -0.018 |
| | | | | ( 0.033 ) | ( 0.033 ) | ( 0.033 ) |
| Health Xp. | | | | 0.308 † | 0.234 | 0.211 |
| | | | | ( 0.170 ) | ( 0.173 ) | ( 0.172 ) |
| DC Pension | | | | | 0.028 | 0.051 |
| | | | | | ( 0.128 ) | ( 0.128 ) |
| DB Pension | | | | | 0.360 ** | 0.376 ** |
| | | | | | ( 0.119 ) | ( 0.119 ) |
| Fin. Wealth | | | | | | 0.349 † |
| | | | | | | ( 0.179 ) |
| Demographix | No | Yes | Yes | Yes | Yes | Yes |

HUSBANDS' Proportional Hazards (Weibull Baseline)

| Variable | Coef. (Std. Err.) | Coef. (Std. Err.) | Coef. (Std. Err.) | Coef. (Std. Err.) | Coef. (Std. Err.) | Coef. (Std. Err.) |
|---|---|---|---|---|---|---|
| $\alpha$ | 1.213 ** | 1.233 ** | 1.233 ** | 1.218 ** | 1.230 ** | 1.230 ** |
| | ( 0.035 ) | ( 0.036 ) | ( 0.036 ) | ( 0.037 ) | ( 0.038 ) | ( 0.038 ) |
| Constant | -5.504 ** | -5.396 ** | -5.341 ** | -5.558 ** | -5.607 ** | -5.614 ** |
| | ( 0.153 ) | ( 0.194 ) | ( 0.220 ) | ( 0.261 ) | ( 0.266 ) | ( 0.265 ) |
| Age Diff. | 0.020 ** | 0.023 ** | 0.023 ** | 0.028 ** | 0.026 ** | 0.027 ** |
| | ( 0.006 ) | ( 0.006 ) | ( 0.006 ) | ( 0.006 ) | ( 0.006 ) | ( 0.006 ) |
| V. G. Health | | | -0.064 | -0.023 | -0.023 | -0.027 |
| | | | ( 0.123 ) | ( 0.128 ) | ( 0.128 ) | ( 0.128 ) |
| Good Health | | | -0.073 | -0.061 | -0.073 | -0.078 |
| | | | ( 0.128 ) | ( 0.133 ) | ( 0.133 ) | ( 0.133 ) |
| Health Ins. | | | | 0.014 † | 0.014 † | 0.014 † |
| | | | | ( 0.007 ) | ( 0.008 ) | ( 0.008 ) |
| Health Xp. | | | | 0.243 † | 0.214 | 0.215 |
| | | | | ( 0.128 ) | ( 0.133 ) | ( 0.134 ) |
| DC Pension | | | | | -0.204 * | -0.206 † |
| | | | | | ( 0.102 ) | ( 0.102 ) |
| DB Pension | | | | | 0.278 ** | 0.278 ** |
| | | | | | ( 0.098 ) | ( 0.099 ) |
| Fin. Wealth | | | | | | 0.084 |
| | | | | | | ( 0.168 ) |
| Demographix | No | Yes | Yes | Yes | Yes | Yes |

WIVES' Simultaneous Duration (Threat point scale=0.6)

| Variable | Coef. (Std. Err.) | Coef. (Std. Err.) | Coef. (Std. Err.) | Coef. (Std. Err.) | Coef. (Std. Err.) | Coef. (Std. Err.) |
|---|---|---|---|---|---|---|
| $\alpha$ | 1.229 ** | 1.238 ** | 1.237 ** | 1.243 ** | 1.245 ** | 1.248 ** |
| | ( 0.029 ) | ( 0.032 ) | ( 0.017 ) | ( 0.011 ) | ( 0.013 ) | ( 0.023 ) |
| $\log(\delta - 1)$ | -3.237 | -3.342 | -3.506 | -3.505 | -3.507 ** | -3.480 ** |
| | ( . ) | ( . ) | ( . ) | ( . ) | ( 1.175 ) | ( 0.597 ) |
| Constant | -5.833 ** | -5.978 ** | -5.792 ** | -6.002 ** | -5.943 ** | -5.985 ** |
| | ( 0.136 ) | ( 0.292 ) | ( 0.354 ) | ( 0.437 ) | ( 0.255 ) | ( 0.354 ) |
| Age Diff. | -0.075 ** | -0.073 ** | -0.067 ** | -0.082 ** | -0.077 ** | -0.079 ** |
| | ( 0.014 ) | ( 0.018 ) | ( 0.015 ) | ( 0.012 ) | ( 0.014 ) | ( 0.012 ) |
| V G Health | | | -0.199 | -0.236 | -0.284 | -0.277 |
| | | | ( 0.278 ) | ( 0.147 ) | ( 0.179 ) | ( 0.252 ) |
| Good Health | | | -0.320 | -0.332 † | -0.400 * | -0.381 |
| | | | ( 0.291 ) | ( 0.181 ) | ( 0.191 ) | ( 0.260 ) |
| Health Ins. | | | | -0.007 | -0.013 | -0.010 |
| | | | | ( 0.066 ) | ( 0.045 ) | ( 0.052 ) |
| Health Xp. | | | | 0.318 | 0.237 | 0.212 |
| | | | | ( 0.266 ) | ( 0.202 ) | ( 0.196 ) |
| DC Pension | | | | | 0.115 | 0.125 |
| | | | | | ( 0.142 ) | ( 0.206 ) |
| DB Pension | | | | | 0.442 * | 0.452 |
| | | | | | ( 0.186 ) | ( 0.277 ) |
| Fin. Wealth | | | | | | 0.399 ** |
| | | | | | | ( 0.153 ) |
| Demographix | No | Yes | Yes | Yes | Yes | Yes |

HUSBANDS' Simultaneous Duration (Threat point scale=0.6)

| **Variable** | **Coef.** (Std. Err.) | **Coef.** (Std. Err.) | **Coef.** (Std. Err.) | **Coef.** (Std. Err.) | **Coef.** (Std. Err.) | **Coef.** (Std. Err.) |
|---|---|---|---|---|---|---|
| $\alpha$ | 1.212 ** | 1.233 ** | 1.233 ** | 1.220 ** | 1.230 ** | 1.230 ** |
| | ( 0.024 ) | ( 0.023 ) | ( 0.026 ) | ( 0.010 ) | ( 0.009 ) | ( 0.018 ) |
| $\log(\delta - 1)$ | -3.123 | -3.455 | -3.381 | -3.455 ** | -3.457 ** | -3.556 ** |
| | ( . ) | ( . ) | ( . ) | ( 0.263 ) | ( 1.179 ) | ( 0.440 ) |
| Constant | -5.501 ** | -5.394 ** | -5.340 ** | -5.557 ** | -5.607 ** | -5.614 ** |
| | ( 0.086 ) | ( 0.117 ) | ( 0.250 ) | ( 0.210 ) | ( 0.200 ) | ( 0.318 ) |
| Age Diff. | 0.023 ** | 0.023 * | 0.023 * | 0.028 ** | 0.027 ** | 0.028 ** |
| | ( 0.008 ) | ( 0.009 ) | ( 0.009 ) | ( 0.007 ) | ( 0.006 ) | ( 0.008 ) |
| V G Health | | | -0.062 | -0.021 | -0.021 | -0.026 |
| | | | ( 0.203 ) | ( 0.136 ) | ( 0.162 ) | ( 0.205 ) |
| Good Health | | | -0.049 | -0.060 | -0.073 | -0.067 |
| | | | ( 0.237 ) | ( 0.110 ) | ( 0.192 ) | ( 0.220 ) |
| Health Ins. | | | | 0.014 | 0.014 | 0.014 |
| | | | | ( 0.013 ) | ( 0.022 ) | ( 0.018 ) |
| Health Xp. | | | | 0.244 | 0.212 † | 0.215 |
| | | | | ( 0.182 ) | ( 0.128 ) | ( 0.188 ) |
| DC Pension | | | | | -0.102 | -0.157 |
| | | | | | ( 0.164 ) | ( 0.146 ) |
| DB Pension | | | | | 0.281 * | 0.278 |
| | | | | | ( 0.126 ) | ( 0.171 ) |
| Fin. Wealth | | | | | | 0.092 |
| | | | | | | ( 0.183 ) |
| Demographix | No | Yes | Yes | Yes | Yes | Yes |

# To Do

- Simulate and check joint retirement patterns implied by estimated parameters.
- Try different auxiliary models.
- For different spouse retirement ages, how does the probability distribution of retirement timing change?

  "In the UK, for instance, the state retirement age for women, which is currently 60 years of age, is set to increase by six months per year from 2010 until it reaches 65 in 2020. (...) Given the incidence of joint retirement in England (...) the question is whether this type of policy will change men's retirement patterns as well." (BBC [2007])