Aging and Pension Reform in a Two-Region World: The Role of Human Capital

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1 Introduction

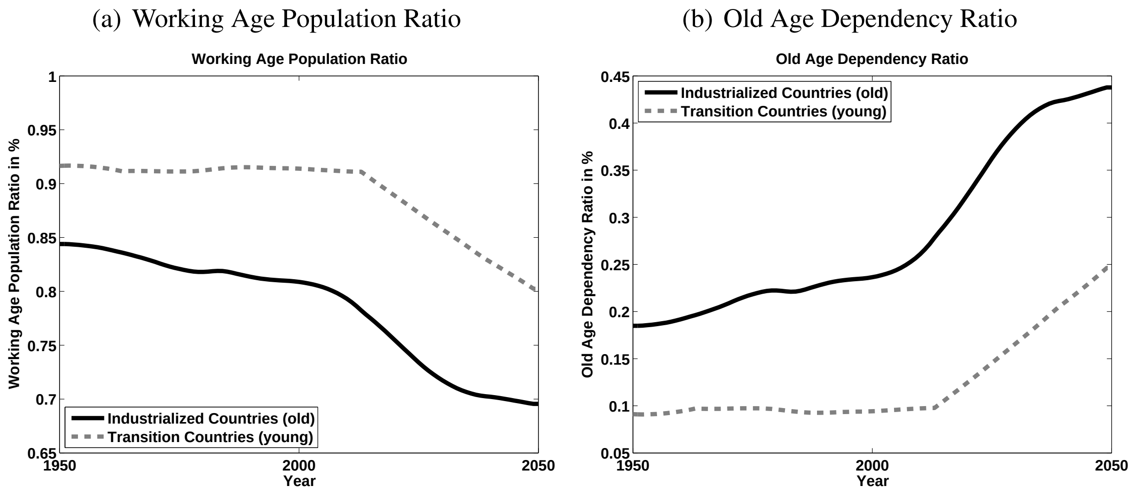
The world will experience major changes in its demographic structure in the next decades. This process is in all countries driven by i) increasing life-expectancy and ii) a decline in birth rates. Timing and extent of this process varies between countries but the economic consequences and the associated challenges are similar everywhere. The fraction of the population in working-age will decrease and the fraction of people in old-age will increase. This process is already well under way in the industrial countries and the fiscal pressure on the public PAYGO pension systems caused by the retirement of the baby-boom generation is a hotly debated issue in public policy. On the contrary, developing countries are relatively young and are projected to face the problems of the demographic transition only in a few decades. This differential population structure will be reflected in different aggregate behavior leading to differences in national savings and investment rates and in a global context to international imbalances in capital and good's flows.

In this paper we start from the insights of standard analysis based on OLG models which predict that demographic change will increase the capital-labor ratio. Hence, rates of return to capital decrease and wages increase, which has adverse welfare consequences for current cohorts who will be retired when the rate of return is low. These detrimental welfare effects may be dampened by two important endogenous margins of adjustment that have received ample attention in the literature. The first adjustment we look at is investing abroad, i.e., openness to international capital markets. Taking the perspective of the group of industrialized countries, we investigate whether the pressure on rates of return can be reduced by international capital flows. The second margin we look at is endogenous human capital formation. Strong incentives to invest in human capital emanate from the combined effects of increasing life expectancy and changes in relative prices induced by demographic change. In general equilibrium, such endogenous human-capital adjustments may substantially mitigate the effects of demographic change on macroeconomic aggregates and individual welfare.

We quantitatively evaluate our findings along these two margins of adjustment in combination with different scenarios of pension policy. We compare a scenario with a constant contribution rate – in which pension payments have to be reduced in order to balance the budget of the social security system – with one with a constant replacement rate – in which the converse adjustments have to take place. We further extend this analysis by a parametric pension reform through which the retirement age is increased. In this scenario, we consider gradual increases of the statutory retirement age by one year when life expectancy increases by 1.5 years. We implement this policy reform for all cohorts who retire after year 2020. Incentives to invest in human capital are strongest when contribution rates are held constant and the retirement age is increased according to this scheme. Our key research question is how welfare of generations who live through the demographic transition is affected by these endogenous adjustments triggered by demographic change and exogenous policy reforms.

Point of departure of our analysis are the key facts of demographic change which we take as an exogenous driving force in our quantitative analysis. The left panel of figure 1 illustrates the impact of demographic change on the working-age population ratio—the ratio of the working-age population (of age 16 – 64) to the total adult population (of age 16 – 90)—and the right panel the old-age dependency ratio—the ratio of the old population of age 65 – 90 to the working age population—in the the major industrial countries and the rest of the world. As the figure shows, the demographic structure is subject to significant changes over time in developed and developing countries (see section 2 for details). The figure shows that there are strong level differences between these two regions but that the overall demographic trends are very similar. To the extent that the group of industrialized countries is relatively open to the rest of the world already today, this implies that changes in relative prices across time are relatively small when we contrast such an open economy scenario with a counterfactual closed economy scenario.

Figure 1: Old Age Dependency Ratio and Working Age Population Ratio



Notes: Data taken from United Nations (2007) and own projections.

In order to quantitatively investigate the relative roles of openness and human capital adjustments for economic aggregates and welfare, we develop a large-scale OLG model in the spirit of Auerbach and Kotlikoff (1987). We extend an otherwise standard model by adding human capital accumulation and an open economy dimension. As the central part of our analysis we work out the quantitative differences between a benchmark model—with two countries and endogenous human capital formation—and counterfactuals models where industrialized countries operate in a closed economy and where human capital may be exogenous.

Our open economy scenario is characterized by an international market for physical capital whereby the market for human capital is local. More precisely, in our exogenous demographic dynamics we allow for (exogenous) migration. These migrants move without human capital.[1] These strong assumptions may be justified on the grounds that much of human capital is country specific.

Our analysis generates a number of interesting findings. We here only focus on the case with fixed contribution rates. First, we find that newborn agents then gain from increasing wages and decreasing returns which relaxes borrowing costs. Expressed as consumption equivalent variations, these welfare gains are between 0.8% and 1.1% of life-time utility. Second, middle-aged and asset rich households experience losses between 6.5% and 3.6% whereby the magnitudes in this range strongly depend on the adjustment of human capital and the retirement age (see below). These losses must be compared with strong welfare gains for all future generations. Third, while openness to international capital markets affects our predictions for per capita GDP and other macroeconomic aggregates, the impact of openness on rates of return to physical capital and wages is relatively small. Consequently, welfare of generations that live through the demographic transition is relatively little affected when we counter-factually assume a closed economy. Fourth, endogenous human capital formation has strong welfare effects. In the open economy, maximum welfare losses of middle-aged households shrink from 6.5% to 4.4% when human capital can endogenously adjust. Fifth, increasing the retirement age has strong welfare effects. Maximum welfare losses of middle-aged households further shrink from 4.4% to 3.6% when the retirement age is increased.

Our work relates to a vast number of papers that have analyzed the economic consequences of population aging and possible adjustment mechanisms. Important examples in closed economies with a focus on social security adjustments include Huang et al. (1997), De Nardi et al. (1999)

[1] For technical reasons we go a step further and assume that all migration takes place before age 16 so that we can treat newborns and immigrants alike.

and, with respect to migration, Storesletten (2000). In open economies, Bösch-Supan et al. (2006), Attanasio et al. (2007) and Krüger and Ludwig (2007), among others, investigate the role of international capital flows during the demographic transition. Further, Domeij and Flodén (2006) show that differences in the population structure significantly affect international capital flows. We add to this literature by highlighting an additional mechanism through which households can respond to demographic change.

Our paper is closely related to the theoretical work on longevity, human capital, taxation and growth[2] and to Fougère and Mérette (1999) and Sadahiro and Shimasawa (2002) who also investigate demographic change in large-scale OLG models with individual human capital decisions. In contrast to their work, we focus our analysis on the relative price changes during the demographic transition and therefore consider an exogenous growth specification.[3] We also extend their analysis along various dimensions. We use realistic demographic projections instead of stylized scenarios. More importantly, our model contains a labor supply-human capital formation-leisure trade-off. It can thus capture effects from changes in individual labor supply, i.e., human capital utilization, on the return of human capital investments. As has already been stressed by Becker (1967) and Ben-Porath (1967) it is important to model human capital and labor supply decisions jointly in a life-cycle framework. Along this line, a key feature of our quantitative investigation, is to employ a Ben-Porath (1967) human capital model and calibrate it to replicate realistic life-cycle wage profiles.[4] Furthermore, we put particular

[2]See for example de la Croix and Licandro (1999), Boucekkine et al. (2002), Kalemli-Ozcan et al. (2000) Echevarria and Iza (2006), Heijdra and Romp (2007) and Ludwig and Vogel (2009). Our paper is also related to a literature emphasizing the role of endogenous human capital accumulation for the analysis of changes to the tax or social security system as in Lord (1989), Trostel (1993), Perroni (1995), Dupor et al. (1996) and Lau and Poutvaara (2006), among others.

[3]Whether the trend growth rate endogenously fluctuates during the demographic transition or is held constant is of minor importance for the questions we are interested in. This is shown in an earlier working paper in a closed economy setup (Ludwig, Krüger, and Börsch-Supan 2007).

[4]The Ben-Porath (1967) model of human capital accumulation is one of the workhorses in labor economics used to understand such issues as educational attainment, on-the-job training, and wage growth over the life cycle, among others, see Browning, Hansen, and Heckman (1999) for a review. More recently, extended versions of the model have been applied to study the

emphasis on the welfare consequences of population aging for households living through the demographic transition.

The paper is organized as follows. In section 2 we present the formal structure of our quantitative model. Section 3 describes the calibration strategy and our computational solution method. Our results are presented in section 4. Finally, section 5 concludes the paper.

2 The Model

We use a large scale multi-country OLG model in the spirit of Auerbach and Kotlikoff (1987) with endogenous labor supply, human capital formation and a standard consumption-saving decision.[5] The population structure is exogenously determined by time and region specific demographic processes for fertility, mortality, and migration, the main driving forces of our model.[6] The world population is divided into 2 regions. We group countries into one of the two regions according to their demographic and economic stage of development. Our first region – which we label "old" – consists of developed nations: USA, Canada, Japan, Australia, New Zealand, Switzerland, Norway and the EU15. The second ("young") region consists of all other countries. In terms of population, the developed world makes up for 23% of the world population in 1950 and its weight decreases to about 14% in the year 2000. The developed word's share in real GDP decreases from 60% in 1950 to 54% in the year 2000 (Maddison 2003).

We choose this simple two-country setup for two reasons. Firstly, the demographic processes *within* the set of countries are rather synchronized. Therefore the intra-regional demo-

significant changes to the U.S. wage distribution and inequality observed since the early 1970s by Heckman, Lochner, and Taber (1998) and Guvenen and Kuruscu (2009).

[5] The model borrows heavily from Ludwig, Schelkle, and Vogel (2011).

[6] Although changes in prices may have – via numerous mechanisms – feedback effects on life expectancy, fertility, and migration we leave these extensions for further research.

graphic differences do not matter for the international capital flows. The quantitatively important capital flows will occur *between* the two regions. Secondly, as our focus is endogenous human capital accumulation, we want to make sure that we have good data for calibrating the model. The availability of data (especially on the individual level) for the "young" countries is a serious restriction on the degree of sophistication for our model. Moreover, the differences between regions are again likely to dominate the differences within regions.

Physical capital is perfectly mobile and can flow across borders whereas human capital (labor) is immobile. Firms produce with a standard constant returns to scale production function in a perfectly competitive environment. Agents contribute a share of their wage to the pension system and retirees receive a share of current net wages as pensions. Technological progress is exogenous.

2.1 Timing, Demographics and Notation

Time is discrete and one period corresponds to one calendar year t. Each year, a new generation is born. Since agents are inactive before they enter the labor market, birth in this paper refers to the first time households make own decisions and is set to real life age of 16 (model age $j = 0$). In the benchmark scenario agents retire at an exogenously given age of 65 (model age $jr = 49$) and live at most until age 90 (model age $j = J = 74$). Both numbers are identical across regions. At a given point in time t, individuals of age j in country i survive to age $j+1$ with probability $\varphi_{t,j,i}$, where $\varphi_{t,J,i} = 0$. The number of agents of age j at time t in country i is denoted by $N_{t,j,i}$ and $N_{t,i} = \sum_{j=0}^{J} N_{t,j,i}$ is total population in t,i.In the demographic projections migration is exogenous and happens at the age of 16. Thus, we implicitly assume that new migrants are born with the initial human capital endowment and human capital production function of the natives. This assumption is consistent with Hanushek and Kimko (2000) who show that individual productivity (and thus human capital) of workers appears mainly to be related to a country's level of schooling and not to cultural factors.

2.2 Households

Each household comprises of one representative agent who decides about consumption, saving, labor supply, and time investment into human capital formation. The remaining time can be consumed as leisure. The household in region i maximizes lifetime utility at the beginning of economic life ($j = 0$) in period t,

$$\max \sum_{j=0}^{J} \beta^j \pi_{t,j,i} \frac{1}{1-\sigma} \{c_{t+j,j,i}^{\phi} (1 - \ell_{t+j,j,i} - e_{t+j,j,i})^{1-\phi}\}^{1-\sigma}, \quad \sigma > 0, \tag{1}$$

where the per period utility function is a function of individual consumption c, labor supply ℓ and time investment into formation of human capital, e. The agent is endowed with one unit of time, so $1 - \ell - e$ is leisure time. β is the raw time discount factor, ϕ determines the weight of consumption in utility and σ is the inverse of the inter-temporal elasticity of substitution with respect to the aggregate of consumption and leisure time. $\pi_{t,j,i}$ denotes the unconditional probability to survive until age j, $\pi_{t,j,i} = \prod_{k=0}^{j-1} \varphi_{t+k,k,i}$, for $j > 0$ and $\pi_{t,0,i} = 1$.

Agents earn labor income (pension income when retired), interest payments on their savings and receive accidental bequests. When working they pay a fraction $\tau_{t,i}$ from their gross wages to the social security system. Net wage income in period t of an agent of age j living in region i is given by $w_{t,j,i}^n = \ell_{t,j,i} h_{t,j,i} w_{t,i} (1 - \tau_{t,i})$, where $w_{t,i}$ is the (gross) wage per unit of supplied human capital at time t in region i. Annuity markets are missing and accidental bequests are distributed by the government as lump-sum payments to the households. The household's dynamic budget constraint is given by

$$
a_{t+1,j+1,i} = \begin{cases} (a_{t,j,i} + tr_{t,i})(1+r_t) + w^n_{t,j,i} - c_{t,j,i} & \text{if } j < jr \\ \\ (a_{t,j,i} + tr_{t,i})(1+r_t) + p_{t,j,i} - c_{t,j,i} & \text{if } j \geq jr, \end{cases} \tag{2}
$$

where $a_{t,j,i}$ denotes assets, $tr_{t,i}$ are transfers from accidental bequests, r_t is the real interest rate, the rate of return to physical capital, and $p_{t,j,i}$ is pension income. Initial household assets are zero ($a_{t,0,i} = 0$) and the transversality condition is $a_{t,J+1,i} = 0$.

2.3 Formation of Human Capital

The key element of our model is endogenous formation of human capital. Households enter economic life with a predetermined and cohort invariant level of human capital $h_{t,0,i} = h_0$. Afterwards, they can invest a fraction of their time into acquiring additional human capital. We adopt a version of the Ben-Porath (1967) human capital technology[7] given by

$$
h_{t+1,j+1,i} = h_{t,j,i}(1-\delta^h_i) + \xi_i(h_{t,j,i}e_{t,j,i})^{\psi_i} \qquad \psi_i \in (0,1), \;\; \xi_i > 0, \;\; \delta^h_i \geq 0, \tag{3}
$$

where ξ_i is a scaling factor, ψ_i determines the curvature of the human capital technology, δ^h_i is the depreciation rate of human capital and $e_{t,j,i}$ is time investment into human capital formation. The parameters of the production function are allowed to vary across countries to allow for region-specific human capital profiles during our calibration period. Since we do not model any other labor market frictions or costs of human capital acquisition this is the only way to replicate the observed differences in the age-wage profiles. However, we adjust the parameters such that they are eventually identical in both regions and thus agents will have c.p. the same life-cycle human capital profile in the final steady-state (see section 3.3).

[7] This functional form is widely used in the human capital literature, cf. Browning, Hansen, and Heckman (1999) for a review.

In this model, the costs of investing into human capital are only the opportunity costs of foregone wage income and leisure. We understand the process of accumulating human capital as a mixture of investment via formal schooling and on the job training programs. Human capital can be accumulated until retirement age but agents optimal investment strategy is such that the time spent on accumulation of human capital endogenously converges to zero some time before retirement.

2.4 The Pension System

The pension system is a simple balanced budget pay-as-you-go system. Agents contribute a fraction $\tau_{t,i}$ of their gross wages and pensioners receive a fraction $\rho_{t,i}$ of the current average net wages of workers.[8] Pensions in each period are then given by $p_{t,j,i} = \rho_{t,i}(1-\tau_{t,i})w_{t,i}\bar{h}_{t,i}$, where $\bar{h}_{t,i} = \frac{\sum_{j=0}^{jr-1} \ell_{t,j,i} h_{t,j,i} N_{t,j,i}}{\sum_{j=0}^{jr-1} \bar{\ell}_{t,j,i} N_{t,j,i}}$ denotes average human capital of workers. Using the formula for $p_{t,j,i}$, the budget constraint of the pension system[9] simplifies to

$$\tau_{t,i} \sum_{j=0}^{jr-1} \ell_{t,j,i} N_{t,j,i} = \rho_{t,i}(1-\tau_{t,i}) \sum_{j=jr}^{J} N_{t,j,i} \quad \forall t. \tag{4}$$

We consider two policy scenarios in order to ensure the long-term sustainablity of the public pension system. In our first scenario, we keep the retirement age constant and adjust either contributions or benefits to balance the system. Our second alternative is a change in the statutory retirement age. In fact, most governments currently implement a mix of our two strategies. In order to highlight the most extreme economic impact of the different reforms, we will perform the two types of policy experiments in isolation.

In our first reforms scenario we keep the retirement at the current level (65 years). Then, we

[8] Pension systems across counties differ along many dimensions. See for instance (Diamond and Gruber 1999) or Whitehouse (2003) for an overview.

[9] The budget constraint is given by $\tau_{t,i} w_{t,i} \sum_{j=0}^{jr-1} \ell_{t,j,i} h_{t,j,i} N_{t,j,i} = \sum_{j=jr}^{J} p_{t,j,i} N_{t,j,i} \quad \forall t.$

either hold the contribution rate constant $\tau_{t,i} = \bar{\tau}_i$ (labeled "const. τ"), and endogenously adjust the replacement rate to balance the budget of the pension system or we hold the replacement rate constant $\rho_{t,i} = \bar{\rho}_i$ (labeled "const. ρ") , and endogenously adjust the contribution rate.

As the second dimension of pension reforms we increase the statutory retirement age. A policy reform along these lines is considered as an alternative option to ease the fiscal pressure of an ageing society on the social security systems. This reform scenario captures two effects on the incentive to acquire human capital: a lengthening of the working life combined with – ceteris paribus – lowering the tax burden on currently working individuals.

2.5 Firms

Firms operate in a perfectly competitive environment and produce one homogenous good according to the Cobb-Douglas production function

$$Y_{t,i} = K_{t,i}^{\alpha}(A_{t,i}L_{t,i})^{1-\alpha}, \tag{5}$$

where α denotes the share of capital used in production. $K_{t,i}, L_{t,i}$ and $A_{t,i}$ are the region-specific stocks of physical capital, effective labor and the level of technology, respectively. Output can be either consumed or used as an investment good. We assume that labor inputs and human capital of different agents are perfect substitutes and effective labor input $L_{t,i}$ is accordingly given by $L_{t,i} = \sum_{j=0}^{jr-1} \ell_{t,j,i} h_{t,j,i} N_{t,j,i}$. Factors of production are paid their marginal products, i.e.

$$w_{t,i} = (1-\alpha)A_{t,i}k_t^{\alpha} \qquad r_t = \alpha k_t^{\alpha-1} - \delta, \tag{6a}$$

$$k_t = k_{t,i} = \frac{K_{t,i}}{A_{t,i}L_{t,i}} \tag{6b}$$

where $w_{t,i}$ is the gross wage per unit of efficient labor, r_t is the interest rate and δ_t denotes the depreciation rate of physical capital. Since we have frictionless international capital markets, capital stocks $k_{t,i}$ adjust such that the rate of return is equalized across regions and are therefore determined by the global capital stock relative to global output (see section on aggregation and equilibrium for more details). Note that since agents and their human capital are immobile by assumption, wages differ across regions and are a function of the country specific productivity $A_{t,i}$. Total factor productivity, $A_{t,i}$, is growing at the region-specific exogenous rate of $g^A_{t,i}$: $A_{t+1,i} = A_{t,i}(1+g^A_{t,i})$.

2.6 Capital Markets

International capital flows and their effect on agent's decisions via changing factor prices are at the heart of our model. Therefore, the modeling assumptions about capital markets will play a crucial role for our results. In order to make the model realistic, we assume that both regions are initially closed economies and model the opening up of capital markets as a non-expected event. In the light of economic history, we believe that this is an appropriate choice. To do so, we first solve for the equilibrium transition path of both economies with agents using only prices and transfers from the closed economy scenario. Then, we "surprise" our agents by opening up the capital markets in 1975. Hence, from 1975 onwards there is only one frictionless capital market and thus the marginal product of capital is equalized across regions.[10] Since we model the opening up as a non-expected (zero probability) event, agents can re-optimize only for their remaining lifetime. Since 1975 is just in the middle of more or less closed capital markets right after WWII and arguably open capital markets at the start of the new millennium, we believe this to be an appropriate choice.

[10] Technically, we store the observed distribution of physical assets, human capital and population for both regions from the closed economy model in the year 1975 and recompute the remaining transition time in the open economy.

2.7 Equilibrium

Denoting current period/age variables by x and next period/age variables by x', a household of age j solves in region i, at the beginning of period t, the maximization problem

$$V(a,h,t,j,i) = \max_{c,e,a',h'} \{u(c, 1-\ell-e) + \varphi_i \beta V(a',h',t+1,j+1,i)\} \tag{7}$$

subject to $w^n_{t,j,i} = \ell_{t,j,i} h_{t,j,i} w_{t,i}(1-\tau_{t,i})$, (2), (3) and the constraint $e \in [0, 1-\ell)$.

Definition 1. *Given the exogenous population distribution and survival rates in all periods* $\{\{\{N_{t,j,i}, \varphi_{t,j,i}\}_{j=0}^{J}\}$ *an initial physical capital stock and an initial level of average human capital* $\{K_{0,i}, \bar{h}_0\}_{i=1}^{I}$*, and an initial distribution of assets and human capital* $\{\{a_{t,0,i}, h_{t,0,i}\}_{j=0}^{J}\}_{i=1}^{I}$*, an approximate competitive equilibrium are sequences of individual variables* $\{\{\{c_{t,j,i}, e_{t,j,i}, a_{t+1,j+1,i}, h_{t+1,j+1,i}\}_{j=0}^{J}\}_{t=0}^{T}\}_{i=1}^{I}$*, sequences of aggregate variables* $\{\{L_{t,i}, K_{t+1,i}, Y_{t,i}\}_{t=0}^{T}\}_{i=1}^{I}$*, government policies* $\{\{\rho_{t,i}, \tau_{t,i}\}_{t=0}^{T}\}_{i=1}^{I}$*, prices* $\{\{w_{t,i}, r_t\}_{t=0}^{T}\}_{i=1}^{I}$*, and transfers* $\{\{tr_{t,i}\}_{t=0}^{T}\}_{i=1}^{I}$ *such that*

1. *given prices, bequests and initial conditions, households solve their maximization problem as described above,*
2. *interest rates and wages are paid their marginal products, i.e.* $w_{t,i} = (1-\alpha)\frac{Y_{t,i}}{L_{t,i}}$ *and* $r_t = \alpha \frac{Y_t}{K_t} - \delta$*,*
3. *per capita transfers are determined by*

$$tr_{t,i} = \frac{\sum_{j=0}^{J} a_{t,j,i}(1-\varphi_{t-1,j-1,i}) N_{t-1,j-1,i}}{\sum_{j=0}^{J} N_{t,j,i}}, \tag{8}$$

4. *government policies are such that the budget of the social security system is balanced every period and region, i.e. equation (4) holds* $\forall t, i$*, and household pension income is given by* $p_{t,j,i} = \rho_{t,i}(1-\tau_{t,i}) w_{t,i} \bar{h}_{t,i}$*,*
5. *regional labor markets clear at the wage rate* $w_{t,i}$*, the world capital market clears at the world interest rate* r_t *and allocations are feasible for all periods:*

$$L_{t,i} = \sum_{j=0}^{jr-1} \ell_{t,j,i} h_{t,j,i} N_{t,j,i} \tag{9a}$$

$$Y_{t,i} = \sum_{j=0}^{J} c_{t,j,i} N_{t,j,i} + K_{t+1,i} - (1-\delta)K_{t,i} + F_{t+1,i} - (1+r_t)F_{t,i}, \tag{9b}$$

$$Y_t = \sum_{i=1}^{I} Y_{t,i} \tag{9c}$$

$$K_{t+1} = \sum_{i=1}^{I} \sum_{j=0}^{J} a_{t+1,j+1,i} N_{t,j,i} \tag{9d}$$

6. *and the sum of foreign assets $F_{t,i}$ in all regions is zero*

$$\sum_{i=1}^{I} F_{t,i} = 0. \tag{10}$$

Definition 2. *A stationary equilibrium is a competitive equilibrium in which per capita variables grow at the constant rate $1+\bar{g}^A$ and aggregate variables grow at the constant rate $(1+\bar{g}^A)(1+n)$.*

2.8 Thought Experiments

The exogenous driving force of our model is the time-varying and region-specific demographic structure. The solution of our model is done in two steps. We firstly assume that both regions are closed and start computations in the year 1750 solving for an artificial initial steady state. We then compute the closed economy equilibrium transition path from 1750 to 2500 when the new steady state is assumed and verified to be reached. Then, we use the distribution of physical capital, human capital and population from the two closed regions in the year 1975 and recompute the equilibrium transition path until the new – open economy – steady state is reached.[11] Finally, we report the simulation results for the main projection period of interest, from 2010 to 2050 . We use data during our calibration period, $1960-2005$, to determine several structural model parameters (cf. section 3).

Our main interest is to compare the time paths of aggregate variables and welfare across two model variants for the different social security reform scenarios for the "old" countries.[12] The first model variant is one with agents adjusting their human capital and in our second model variant we keep the human capital profile constant across cohorts. Therefore, our strategy is to first solve and calibrate for the transitional dynamics in the open economy using the model as described above. Then, we use the endogenously generated human capital profile to compute

[11] In fact, for both versions changes in variables which are constant in steady state are numerically irrelevant already around the year 2350.

[12] Results for the "young" countries available upon request.

average time investment and the associated human capital profile which is then used in the alternative model with fixed productivity profile. We perform the computation for the different social security policy scenarios described in subsection 2.4. Time spent for human capital investment is computed as $\bar{e}_{j,i} = \frac{1}{t_1 - t_0 + 1} \sum_{t=t_0}^{t_1} e_{t,j,i}$ for our calibration period. In the alternative model, we impose the constraint $e_{t,j,i} = \bar{e}_{j,i}$. and obtain the human capital profile from (3) by iterating forward on age.[13]

3 Calibration and Computation

To calibrate the model, we choose model parameters such that simulated moments match selected moments in the data and the endogenous wage profiles match the empirically observed wage profile during the calibration period $1960-2005$.[14] The calibrated parameters are summarized in table 1 below and we provide a detailed discussion of our calibration strategy and calibration targets below.

3.1 Demographics

Actual population data from $1950-2005$ are taken from the United Nations (2007). For the period until 2050 we use the same data source and choose the UN's "medium" variant for the fertility projections. However, we have to forecast population dynamics beyond 2050 to solve our model. The key assumptions of our projection are as follows: First, for both regions total fertility rate is constant at 2050 levels until 2100. Then we adjust fertility such that the number of newborns is constant for the rest of the simulation period. Second, we use the life expectancy

[13]We restrict the time investment into human capital production to be identical for all cohorts (instead of using each cohort's own endogenous profile and keeping it fixed). We do this in order not to change the time endowment available to the household from cohort to cohort. Thus, we set the time endowment for each cohort to $1 - \bar{e}_j - \ell_j$.

[14]We perform this moment matching in the endogenous human capital model and the constant contribution rate scenario for the benchmark pension system. We do not re-calibrate model parameters across social security scenarios or for the alternative human capital model, mainly because any parametric change would make comparisons (especially welfare analysis) across models impossible.

forecasted by the United Nations (2007) and extrapolate it until 2100 at the same (region and gender-specific) linear rate.[15] Then we assume that life expectancy in the "old" nations stays constant. Life expectancy in the rest of the world keeps rising until it reaches the level in the industrialized countries by the year 2300. This choice ensures that in the final steady state, the population structure is identical across world regions. By delaying this adjustment process to 2300 we make sure that on the one hand we exclude any anticipation effects of currently living generations, and on the other hand we have enough periods left to test the convergence properties of our model. These assumptions imply that a stationary population is reached in about 2200 in the "old" nations and in 2300 in the rest of the world.

3.2 Households

The parameter σ, the inverse of the inter-temporal elasticity of substitution, is set to 2. The time discount factor β is calibrated to match the empirically observed capital-output ratio of 2.8 in the North which requires $\beta = 0.98$. To calibrate the weight of consumption in the utility function, we set $\phi = 0.37$ by targeting an average labor supply of 1/3 of the total available time.[16] As usual in the literature, we constrain the parameters of the utility function to be identical across regions.

[15]Life expectancy estimated by the UN for cohort born in 2050 is in the industrialized nations 81.5 year for men and 86.8 year for women. In the rest of the world, life expectancy is 71.7 for men and 75.7 for women. The estimates of the trend are as follows: in the industrialized countries life expectancy at birth increasers for each cohort at a linear rate of 0.12 years for men and 0.117 years for women. For the rest of the world the slope coefficient for is 0.204 for men and 0.217 for women. See also Oeppen and Vaupel (2002) for the evolution of life expectancy.

[16]The time series we use for calibration is – due to our assumption about the opening-up – partly from the closed economy and partly from the open economy. In the calibration procedure we target moments only for the "old" region, the main reason being the lack of reliable data.

3.3 Individual Productivity and Labor Supply

We choose values for the parameters of the human capital production function such that average simulated wage profiles resulting from endogenous human capital formation replicate empirically observed wage profiles. Data for age specific productivity are taken from Huggett et al. (2007).[17] We first normalize $h_0 = 1$, and then determine the values of the structural parameters $\{\xi_i, \psi_i, \delta_i^h\}_{i=1}^I$ by indirect inference methods (Smith 1993; Gourieroux et al. 1993). To this end we run separate regressions of the data and simulated wage profiles on a 3rd-order polynomial in age given by

$$\log w_{j,i} = \eta_{0,i} + \eta_{1,i} j + \eta_{2,i} j^2 + \eta_{3,i} j^3 + \varepsilon_{j,i}. \tag{11}$$

Here, $w_{j,i}$ is the age specific productivity. Denote the coefficient vector determining the slope of the polynomial estimated from the actual wage data by $\overrightarrow{\eta}_i = [\eta_{1,i}, \eta_{2,i}, \eta_{3,i}]'$ and the one estimated from the simulated average human capital profile of cohorts born in 1960 – 2005 by $\overrightarrow{\hat{\eta}} = [\hat{\eta}_{1,i}, \hat{\eta}_{2,i}, \hat{\eta}_{3,i}]'$. The latter coefficient vector is a function of the structural model parameters $\{\xi_i, \psi_i, \delta_i^h\}_{i=1}^I$. Finally, the values of our structural model parameters are determined by minimizing the distance $\|\overrightarrow{\eta}_i - \overrightarrow{\hat{\eta}}_i\| \ \forall i$, see subsection 3.6 for further details.

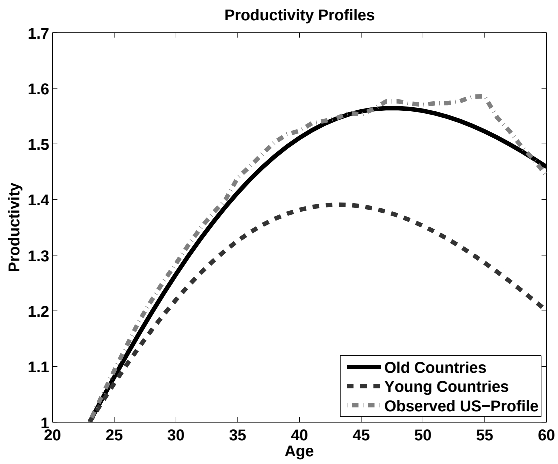
Figure 2 presents the empirically observed productivity profile and the estimated polynomials for the different regions. Our coefficients for the "young" countries[18] and the shape of the wage profile are in line with others reported in the literature, especially with those obtained by Hansen (1993) and Altig et al. (2001). The estimate of δ^h of 1.4% for developing and 0.9% for developed countries is in a reasonable range (Arrazola and de Hevia (2004), Browning, Hansen, and Heckman (1999)), and the estimate of $\psi \approx 0.60$ is also in the middle of the range

[17] We thank Mark Huggett for sending us the data.

[18] The coefficient estimates from the data are η_0: -1.6262, η_1: 0.1054, η_2: -0.0017 and η_3: 7.83e-06. We do not display the calibrated profiles in figure 2 because they perfectly track the polynomial obtained from the data.

reported in Browning, Hansen, and Heckman (1999). Due to lack of reliable individual wage data or good estimates for age-wage profiles we cannot apply the same technique to developing countries. Instead, we take the polynomial estimated on the U.S.-profile and scale the coefficient η_1 by a factor of 0.95. The resulting age-wage profile corresponds then to a profile estimated on the Mexican data by Attanasio, Kitao, and Violante (2007). The main difference between the two profiles is wages in the U.S. drop by 10% and Mexican wages by 20% from their peak to retirement age and that the maximal wage in the U.S. is about 100% higher than the wage at entry into the labor market. The same number in Mexico is about 90%. They attribute the differences (US profiles are steeper and drop less towards the end of working life) to differences in the physical requirements in the two economies. Working in the probably less human-capital intensive Mexican labor market requires relatively more physical strength which is likely to reach its peak earlier and decrease faster afterwards.

Figure 2: Wage Profiles



Notes: Data standardized by the wage at the age 23. Source: Huggett et al. (2007) and own calculations.

To minimize biases, we adjust the parameters of the human capital production function such that they are eventually identical in both regions. To this end we parameterize the adjustment path and calibrate it such that parameters start to change for the cohort born in the year 2100 and are identical for the cohort bon in the year 2300. We denote the vector of parameters $\{\xi_i, \psi_i, \delta_i^h\} = \vec{\chi}_i$ and choose the functional form

$$\vec{\chi}_{i,k} = \vec{\chi}_{i \neq j,k} + \Delta(\chi_{j,k}) \cdot t \quad k = 1,2,3, \tag{12}$$

for the adjustment process where $\Delta(\chi_{j,k})$ denotes the per period linear adjustment of the parameter, t is the length of the adjustment period, and k is an element from χ_i. Thus, the parameters of the human capital accumulation technology are initially different and are then adjusted over time such that the two vectors are eventually identical.

3.4 Production

We calibrate the capital share in production, α, to match the income share of labor in the data which requires $\alpha = 0.33$. The average growth rate of total factor productivity, $\bar{g}_i^A$, is calibrated such that we match the region-specific growth rate of GDP per capita, taken from Maddison (2003). Growth of output per capita in the "old" countries during our calibration period is 2.8%. Accordingly, we set growth rate of TFP to 1.85% to meet our calibration target. To match the observed growth of GDP per capita of 2.2% in the "young" countries, we let TFP grow at a rate of 1.5% From 2100 onwards we let the growth rate of TFP in the "young" countries adjust smoothly to the growth rate in the "old" countries. This adjustment process is assumed to be completed in 2300. Further, we compute relative GDP per capita from Maddison (2003) for both regions in 1950 and use this ratio to calibrate the relative productivity levels at the beginning of the calibration period. Initially, per capita GDP in the developing countries is only 20% of income per capita in the industrialized nations. Finally, we calibrate δ such that our simulated data match an average investment output ratio of 20% in the North which requires $\delta = 0.035$.

3.5 The Pension System

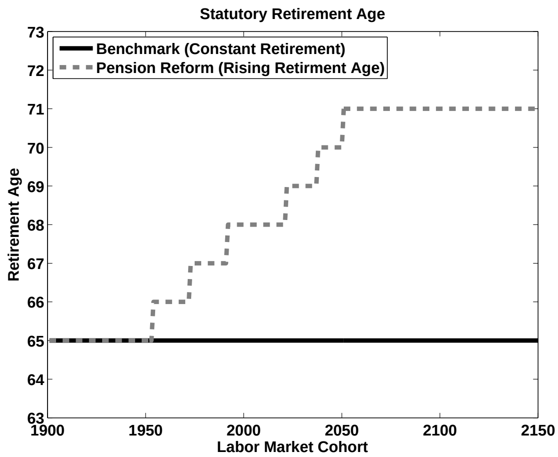
In our first social security scenario ("const. τ") we fix contribution rates and adjust replacement rates of the pension system. Since there is no yearly data on contribution rates for sufficiently many countries, we use the data from Palacios and Pallarés-Miralles (2000) for the mid 1990s and assume that the contribution rate was constant through the entire calibration period. To be more precise, on the individual county level we use the pension tax as a share of total labor costs weighted by the share of contributing workers to compute a national average. Then we weight these number by total GDP to compute a representative number for the two world regions. The contribution rate in the "young" ("old") region is then 4.1% (10.9%). Given the initial demographic structures, the replacement rate is 13.8% (20.4%) in the "young" ("old") region. In our baseline social security scenario we freeze the contribution rate at the level used for the calibration period for all following years. When simulating the alternative social security scenario with constant replacement rates ("const. ρ") we feed the equilibrium replacement rate obtained in the "const. τ" scenario into the model and hold it constant at the level of the year 2000 for all remaining years.[19] Then, the contribution rate endogenously adjusts to balance the budget of the social security system. In both scenarios we assume that the retirement age is fixed at 65 years and agents do not expect any change. We label this scenario as "Benchmark" ("BM") in the following figures.

For the second type of policy reform we increase the retirement age by linking the new retirement age to the remaining life expectancy at age 65 (the current retirement age). We assume that for an increase in life expectancy by 1.5 years, retirement increases by one year.

[19] The choice of the year 2000 was motivated by the fact that this is the last year for which we had data on social security.

We model this change – labeled "Pension Reform" ("PR") – by assuming that this reform affects already workers being the labor market in 1955 (birth cohort 1939) by raising their retirement age immediately by one year and applying the rule from above for all following cohorts.[20] Therefore, the exogenous increase in the number of workers will occur in the year 2021. This reform has direct effect via lengthening expected lifetime labor supply of workers and changing prices for retirees. Given our projections of life expectancy, the retirement age will eventually settle down at 71 years, a value also discussed in the public debate about pension reforms. We show the stepwise increase in the retirement age in figure 3.

Figure 3: Retirement Age



Notes: The jumps in the broken line indicate the cohort which is affected by the change in the retirement age and not the actual time when the number of workers is increasing.

3.6 Computational Method

For a given set of structural model parameters, solution of the model is by outer and inner loop iterations. On the aggregate level (outer loop), the model is solved by guessing initial time paths of the following variables: the capital intensity, the ratio of bequests to wages, the replacement

[20]We also simulated a pension reform by applying the formula from above for the cohort entering the labor market 1955 *without* immediately increasing the retirement age. Given the dynamics of the life expectancy this reform only has an effect on the cohort entering the labor market in 2007 and has therefore negligible effects for the period 2010-2050.

Table 1: Model Parameters

			"Young"	"Old"
Preferences	σ	Inverse of Inter-Temporal Elasticity of Substitution	2.00	
	β	Pure Time Discount Factor	0.985	
	ϕ	Weight of Consumption	0.370	
Human Capital	ξ	Scaling Factor	0.176	0.166
	ψ	Curvature Parameter	0.576	0.586
	δ^h	Depreciation Rate of Human Capital	1.4%	0.9%
	h_0	Initial Human Capital Endowment	1.00	1.00
Production	α	Share of Physical Capital in Production	0.33	
	δ	Depreciation Rate of Physical Capital	3.5%	
	g^A	Exogenous Growth Rate		
		Calibration Period	1.5%	1.9%
		Final Steady State	1.9%	1.9%

Notes: "Young" and "Old" refer to the region. Only one value in a column indicates that the parameter is identical for both regions.

rate (or contribution rate) of the pension system and the average human capital stock for all periods from $t = 0$ until T. For the open economy we impose the restriction of identical capital intensity for both regions but require all other variables from above to converge for each country separately. On the individual level (inner loop), we start each iteration by guessing the terminal values for consumption and human capital. Then we proceed by backward induction and iterate over these terminal values until convergence of the inner loop iterations. In each outer loop, disaggregated variables are aggregated each period. We then update aggregate variables until convergence using the Gauss-Seidel-Quasi-Newton algorithm suggested in Ludwig (2007).

To calibrate the model in the "const. τ" scenario, we consider additional "outer outer" loops to determine structural model parameters by minimizing the distance between the simulated average values and their respective calibration targets for the calibration period 1960 – 2005. To summarize the description above, parameter values determined in this way are β, ϕ, δ, and $\{\xi_i, \psi_i, \delta_i^h\}$.

4 Results

We will divide the description of our results along two dimensions. In the first part we look at the evolution of selected macroeconomic variables during the period 2010-2050. Within that part, we fill first describe the results obtained from our benchmark calibration with the retirement age 65 years. We report the results for the model variants with endogenous and exogenous human capital profiles in open and closed economies. The results obtained in this section should serve as a benchmark to evaluate the effects of the two pension reforms with increases in the statutory retirement age. The second section will then mainly emphasize effects of more radical pension reforms relative to only adjusting contribution or replacement rates.

The second part of this section will focus on the welfare effects of aging. The main strategy is closely follows the first part: we first analyze the welfare effects separately for the closed and open economy using our benchmark pension system. Then we compare the welfare effects of rising retirement age to the results obtained from our benchmark model.

4.1 Transitional Dynamics

We divide our analysis of the transitional dynamics into two parts. In the first part we look at the evolution of economic aggregates such as the rate of return and detrended GDP. When we evaluate future trends of economic aggregates in this first part, we do so in two steps.

In our first step, we look at a comparison of open and counterfactual closed economy versions of our model for two pension scenarios (a "fixed contribution rate" and a "fixed replacement rate" system) and two human capital scenarios ("exogenous" versus "endogenous" human capital). What we learn from this exercise is that openness of the economy matters for the evolution of aggregates such as GDP per capita but, probably surprisingly, not so much for rates of return. What matters more for the time path of the latter is whether human capital is endogenous and how the pension system is designed.

In our second step, we evaluate how our findings are affected by increasing the retirement age. We label this as "pension reform" ("PR"). Given our findings from the first step, we focus our analysis only at the open economy model / endogenous human capital version of our model. We find that increasing the retirement age, by increasing labor supply, will significantly alter the time path of future rates of return to capital and wages.

These insights – namely that endogenous formation of human capital and the design of the pension system are key for the future time paths of wages and returns – are important for our analysis in the second part in which we evaluate welfare of households who live through the demographic transition. We find that, when the contribution rate is held constant, increasing wages dominate for newborn households who hence experience welfare gains whereby the converse applies to old and asset rich households. Gains of the young (and losses of the old) are significantly higher (lower) when human capital can endogenously adjust and when the retirement age increases. On the contrary, the difference between our closed and open economy scenarios is found to be not that big.

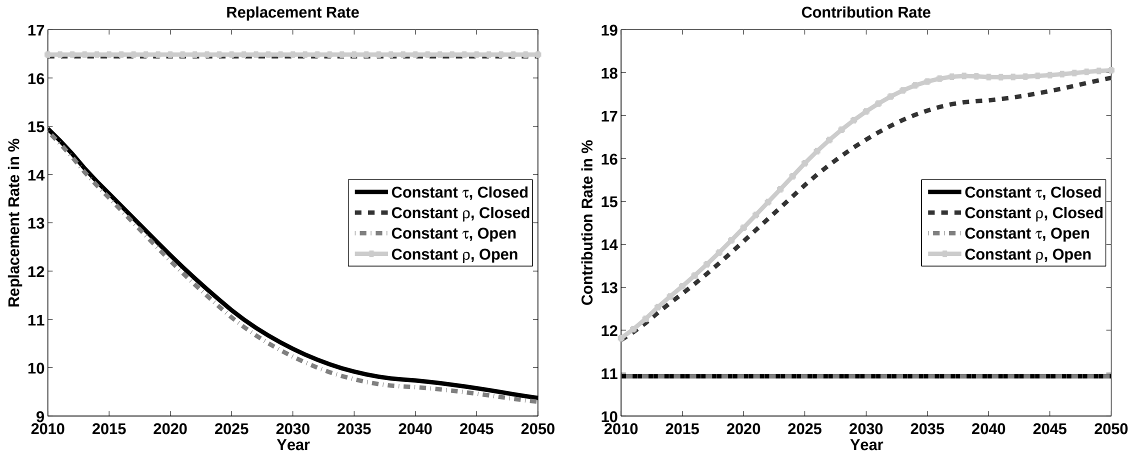
4.1.1 Part I: Macroeconomic Aggregates

Aggregate Variables for the Benchmark Model

Figures 4(a) and 4(b) depict the evolution of the contribution and replacement rates for the benchmark pension system. Holding the replacement rate constant at 10.9% requires an increase of the contribution rate to 18% in 2050. Conversely, keeping the replacement rate unchanged during the entire period at 16.4% requires a drop in the replacement rate to 9.4% until 2050. Small differences emanate in the graphs for the closed and open economy scenarios of our model. These are induced by differential paths of wages and labor supply as well as human capital formation in the respective model variants.

Figure 4: Adjustment of Pension System, Benchmark Pension System

(a) Constant Replacement Rate (b) Constant Contribution Rate



Notes: "Open" and "Closed" refer to the results obtained from the closed and open economy versions. All results obtained from the endogenous human capital scenario, benchmark pension system with constant retirement age ("BM")

The evolution of the most important macroeconomic variables for the scenario with constant replacement rate ("const. ρ") is presented in figure 5. Figure 6 contains the same results but for the alternative adjustment of the pension system where we adjust the replacement rate ("const. τ").

Figures 5(a) and 6(a) show the evolution of the rate of return to physical capital for different model variants. Looking at the development of the rate of return in a closed economy we observe the well known result of falling returns due to population aging. However, we also observe that the drop in the interest rate is much lower in the model where human capital is endogenous as opposed to the "standard" model with exogenously given life-cycle productivity profile. This effect is the result of higher investment into human capital due to falling returns. Therefore, the accumulation of physical capital is lower which dampens the fall in the interest rate. In the open economy variant, we observe a qualitatively similar result with two differences. As “young” economy is importer of capital, the interest rate in the open economy is initially higher than in the closed economy. This pattern is reversed after year 2030 when baby boomers retire in the “old” economy and withdraw their funds (see below). Quantitatively, initial differences are relatively small when human capital formation is endogenous.

We next report the level of de-trended GDP per capita in figure 5(d) and 6(d) where we have standardized the value for the year 2010 to 100. In both model versions with endogenous human capital the income level in 2050 is by an economically significant amount above the level of the “standard” model. This effect is due to more investment into human capital. Turning to the comparison between closed and open economies we observe that income in 2050 in the open economy tends to be higher compared to the closed economy (for endogenous and exogenous human capital).

GDP per capita in the closed economy is initially higher than in the open economy. The reason is the inflow and outflow of physical capital during the transition period. As can be seen from figures 5(b) and 6(b), the “old” region has a positive net foreign asset position until 2035 (2040) in the version with endogenous (exogenous) human capital. This initial outflow of capital decreases production at home and thereby GDP per capita. However, as the capital flows are reversed and capital is repatriated, GDP per capita is surpassing the level of the closed economy.

These differences between the open and closed economy scenarios are most pronounced when human capital adjusts. This can be seen in figures 5(c) and 6(c). Average human capital is higher in the open economy compared to the closed economy after 2035. The initially lower time investment into human capital accumulation in the open economy scenario can be attributed to the lower wage growth and higher rate of return compared to the closed economy scenario. In the closed economy, the demographic structure of the "old" economy leads to relatively more pronounced accumulation of physical capital which can be only used domestically. Therefore, opening up the economy and thereby exporting some of the physical capital to regions with higher returns is initially detrimental for investment into human capital at home.

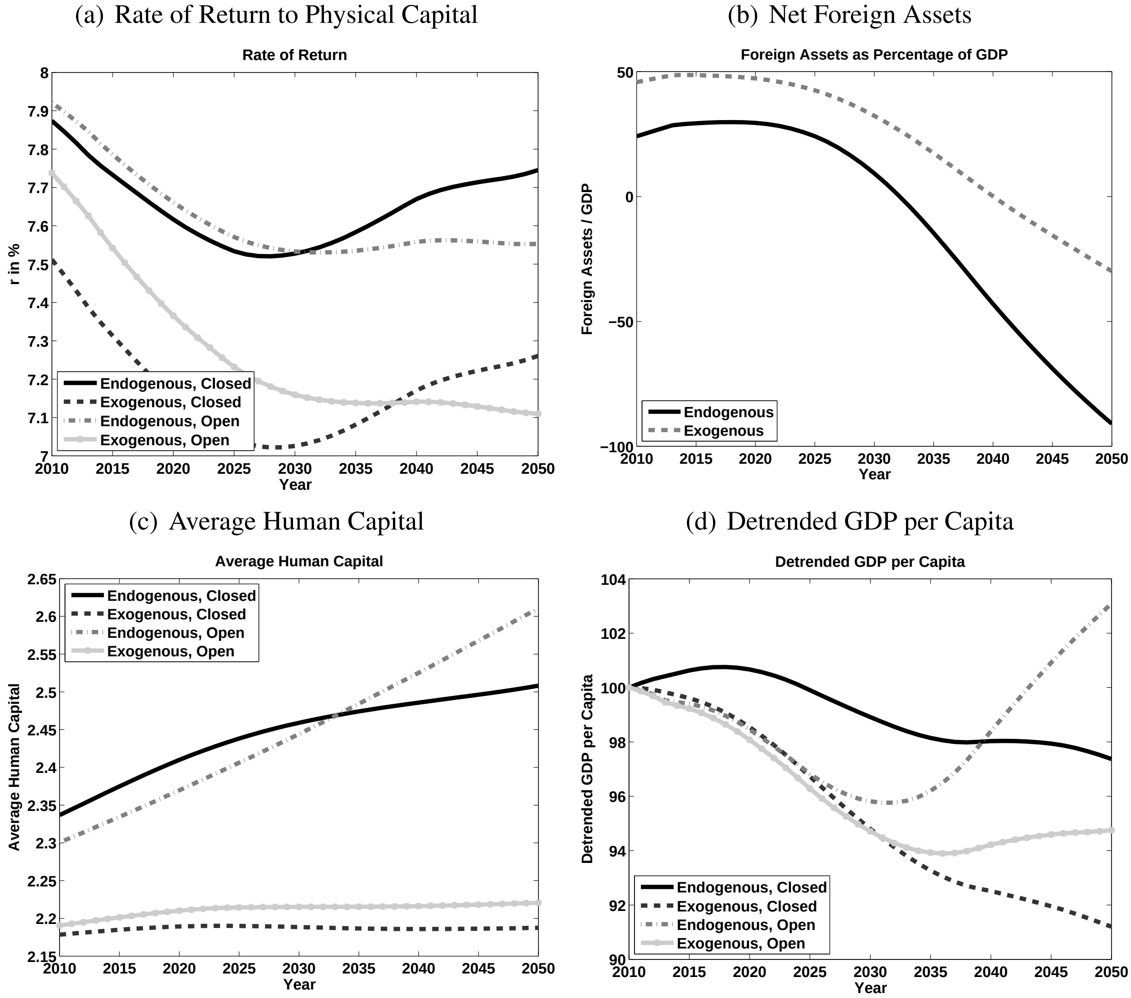
Regarding our comparison across pension scenarios, it is striking to observe that, in case human capital adjusts, detrended per capita GDP increases rather than decreases when contribution rates are held constant. Endogenous human capital formation in a combination with a constant contribution rate in the pension system hence pushes growth rates along the transition above the long-run trend level of 1.8 percent despite the seemingly detrimental effects of aging on raw labor supply.

Aggregate Variables for the Pension Reform Scenario

We now investigate how our results are affected once we let the retirement age increase according to the pattern shown in figure 3. To focus our analysis, we do so in the our benchmark open economy model with endogenous human capital formation. We continue with our comparison across the two pension designs – constant contribution and constant replacement rates – which are subject to policy changes in the retirement age. To distinguish increases in retirement ages semantically from the aforementioned scenarios, we label the experiment of increasing the retirement age as a "pension reform".

Figure 7 displays the associated time paths of replacement and contribution rates. Observe that, in case of a fixed contribution (replacement) rate, the base replacement (contribution) rate is higher when the retirement age adjusts. This is so for mainly two reasons. First, there

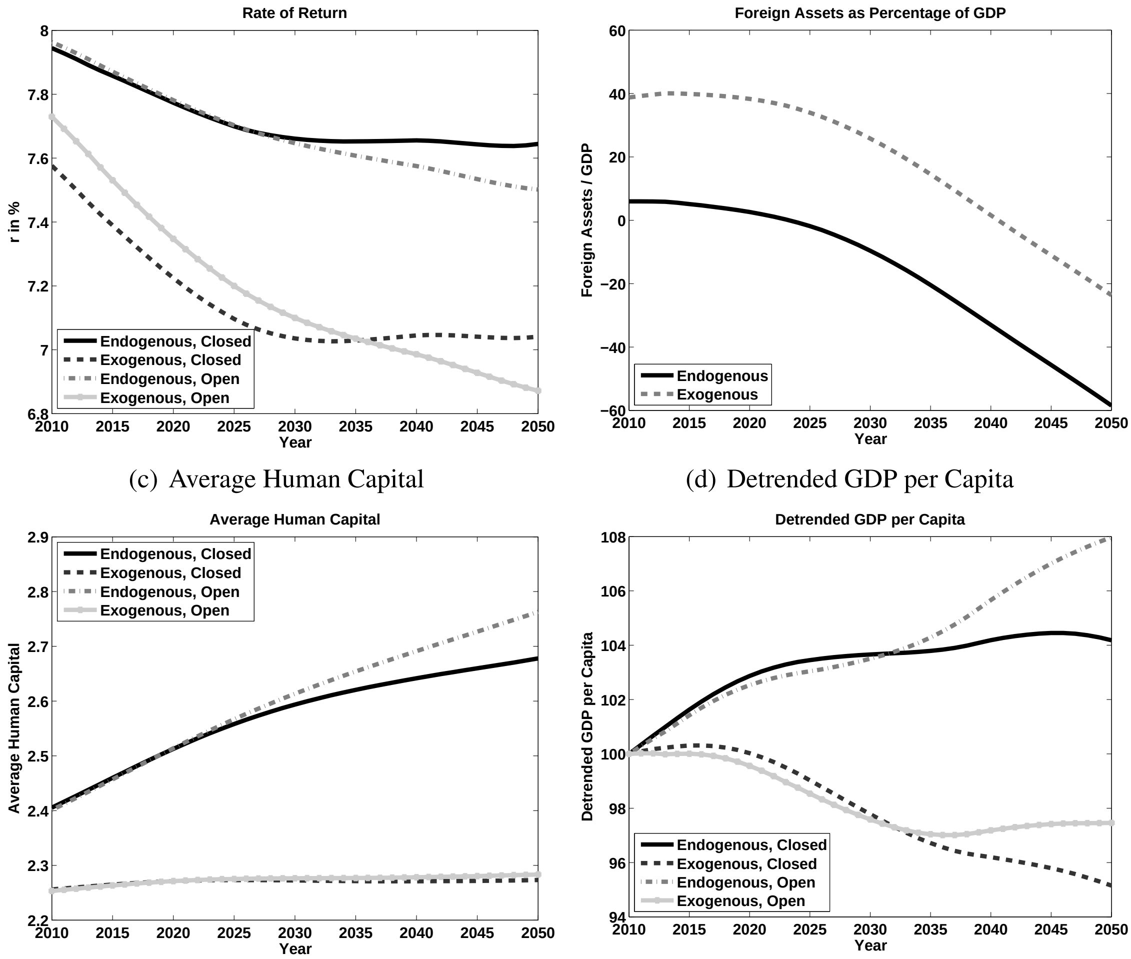
Figure 5: Aggregate Variables for Constant Replacement Rate Scenario, Benchmark Pension System



Notes: "Endogenous" and "Exogenous" refer to the human capital production profile. "Open" and "Closed" refer to the results obtained from the closed and open economy versions. All results obtained from the constant replacement rate scenario, benchmark pension system with constant retirement age ("BM").

is a higher exogenous amount of raw labor in the economy. Second, as shown below, higher retirement ages increase the incentive to invest in human capital over the life-cycle. Hence, also the quality of labor is higher when retirement ages are raised. The figure also displays jumps in the relevant variables. This is due to the fact that our smallest unit of time is one calendar year and it is therefore not possible to implement more gradual changes in the retirement age leading to smoother transitions.

Figure 6: Aggregate Variables for Constant Contribution Rate Scenario, Benchmark Pension System



(a) Rate of Return to Physical Capital

(b) Net Foreign Assets

(c) Average Human Capital

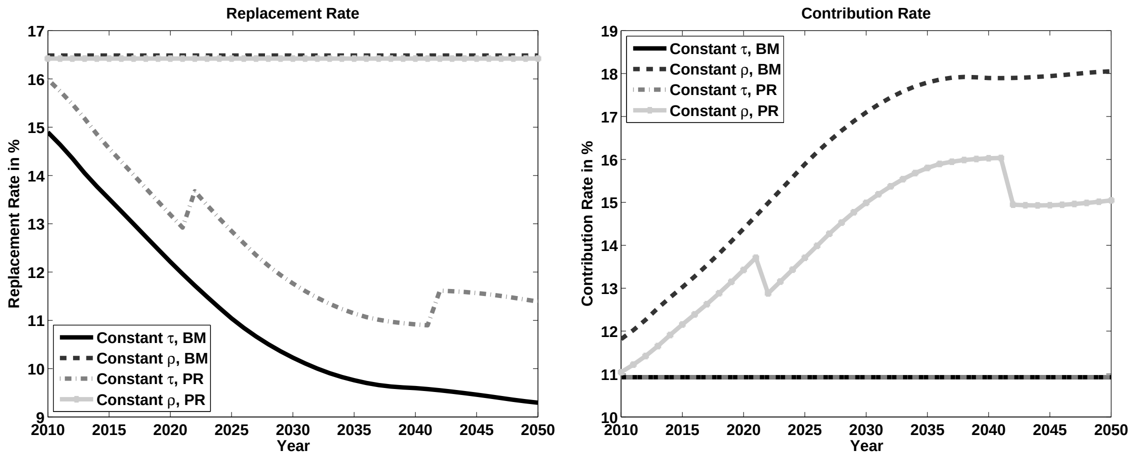
(d) Detrended GDP per Capita

Notes: "Endogenous" and "Exogenous" refer to the human capital production profile. "Open" and "Closed" refer to the results obtained from the closed and open economy versions. All results obtained from the constant contribution rate scenario, benchmark pension system with constant retirement age ("BM".)

Our results of this experiment on macroeconomic variables are summarized in figure 8. Increasing the retirement age has strong effects. The level of rates of returns are significantly higher than in our benchmark scenario. This is so because of the aforementioned effects of increasing the retirement age on total effective labor supply. As a consequence, even when replacement rates are held constant – and contribution rates have to adjust correspondingly – GDP per capita is now found to increase.

Figure 7: Adjustment of Pension System, Pension Reform

(a) Constant Replacement Rate (b) Constant Contribution Rate



Notes: All results obtained from the model with endogenous human capital and open capital markets.

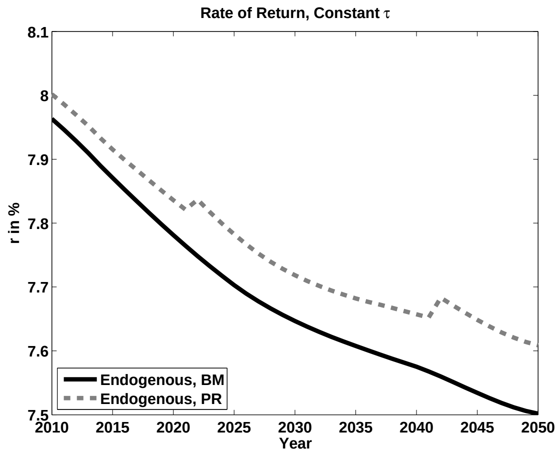
4.1.2 Part II: Welfare Effects

In our model, households are affected by two distinct consequences of demographic change. First, for given prices utility increases because their survival probabilities increase. Second, households are affected by changes in prices and transfers due to the general equilibrium effects of aging. For cohorts currently alive, these profound changes can have – depending on the position in the life-cycle – positive or negative welfare effects. A third effect comes into play when the retirement age is increased. Then, we "force" agents to work longer which reduces leisure time and thereby lifetime utility. At the same time, increasing the retirement age increases the net present value of a household resources which increases utility. Furthermore, as shown above, increasing the retirement age leads to higher levels of rates of return (lower wages) and, as displayed in figures 8(b) and 8(b), lower decreases of the rate of return as societies are aging.

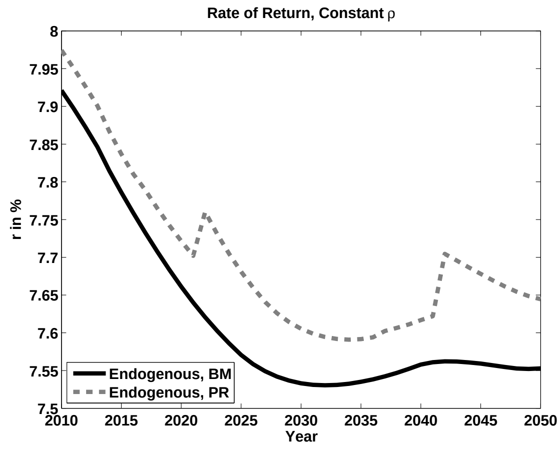
We now want to isolate the effect of changing prices, taxes and transfers on households' lifetime utility. To this end, we first compute the (remaining) lifetime utility of an agent of age j

Figure 8: Aggregate Variables, Endogenous Human Capital in Open Economies, Pension Reform

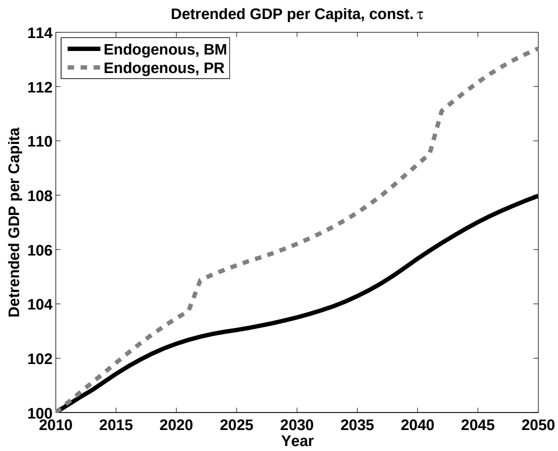
(a) Rate of Return to Physical Capital, Constant τ



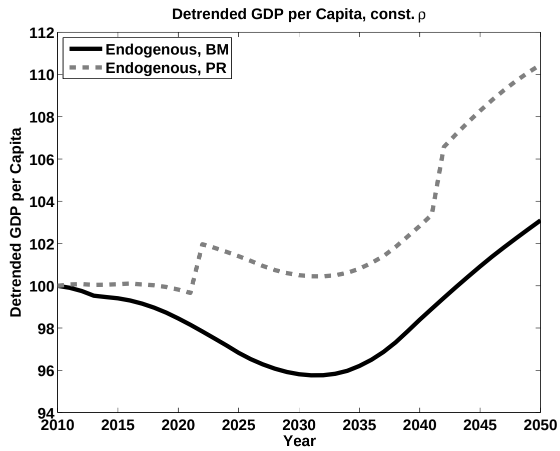
(b) Rate of Return to Physical Capital, Constant ρ



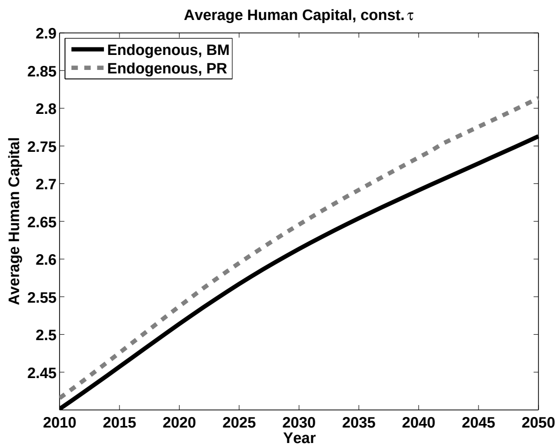
(c) Detrended GDP per Capita, Constant τ



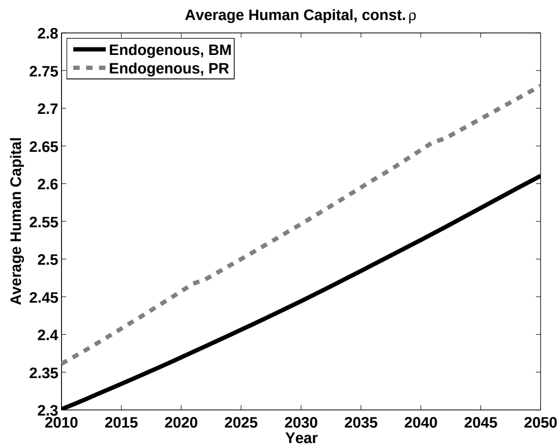
(d) Detrended GDP per Capita, Constant ρ



(e) Average Human Capital, Constant τ



(f) Average Human Capital, Constant ρ



Notes: All results obtained from the model with endogenous human capital and open capital markets.

born in year t using the full set of (time varying) general equilibrium prices, taxes and transfers. Then, we hold all prices and transfers constant at their value from the year 2010 and recompute the agent's remaining lifetime utility. For both scenarios we keep the survival probabilities at their values from the year 2010 constant. Following the convention in the literature, we compute the consumption equivalent variation $g_{t,j,i}$, i.e. the percentage of consumption that needs to be given to the agent at each date for her remaining lifetime at prices from 2010 in order to make her indifferent between the two scenarios. Positive values of $g_{t,j,i}$ thus indicate welfare gains from the general equilibrium effects of aging.[21]

In the remainder of this section we stick to the structure from the previous section. We first report the numbers from our welfare analysis for agents living in the benchmark pension scenario holding the retirement age constant. Then we advance to the comparison of the effects of increasing the retirement age. In so doing, to work out the distributional consequences across generations, we first look at the welfare consequences for agents alive in 2010, followed by an analysis of welfare of future generations.

Welfare of Generations Alive in 2010 - Benchmark Model

The analysis in this section performs an inter-generational welfare comparison of the consequences of demographic change holding the retirement age at the current level. The results for the "const ρ" ("const τ") scenario are shown in figure 9(a) (9(b)). They can be summarized as follows: for constant contribution rates, newborns in the endogenous (exogenous) model benefit up to 0.7% (0.4%) of lifetime consumption from the general equilibrium effects of aging. This is due to increasing wages and decreasing interest rates. Higher wage growth makes

[21]Using the functional form from equation (1) the consumption equivalent variation is given by $g_{t,j,i} = \left(\frac{V_{t,j,i}}{V_j^{2010}}\right)^{\frac{1}{\phi(1-\sigma)}} - 1$ where $V_{t,j,i}$ denotes lifetime utility using general equilibrium prices and V_j^{2010} is lifetime utility using constant prices from 2010.

investment into human capital more attractive and falling interest rates decrease the costs of borrowing. Middle aged agents with high levels of physical assets incur welfare losses due to falling interest rates. Further, their ability to change human capital investment is restricted as they are at a relatively advanced stage in their life-cycle. Retired agents additionally incur losses from falling pensions (due to constant contribution rates).

Of particular interest is that agents in the endogenous human capital investment model incur much lower losses compared to households with a fixed human capital profile. With endogenous human capital, maximum losses are about 32% lower compared to the model with a fixed human capital profile. Thus, agents with the possibility to react to changing prices will do so and this decreases their potential losses by a considerable amount.

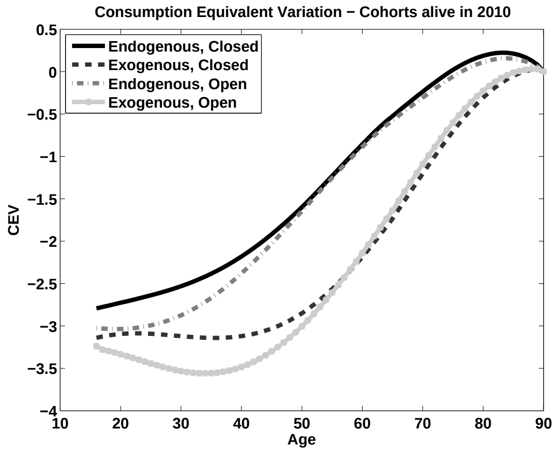
On the contrary, openness per se does not have strong effects on welfare. This is directly related to our insights from the previous part where we have shown that the time paths of rates of return to physical capital (and thereby wages) do not differ much between our open and our counterfactual closed economy scenario.

In the constant replacement rate scenario the majority of the population loses from the effects of demographic change. Young agents are worse off as constant replacement rates require rising contributions which depresses net wages and incentives to invest in human capital. Very old agents experience small gains in lifetime utility due to rising pensions.

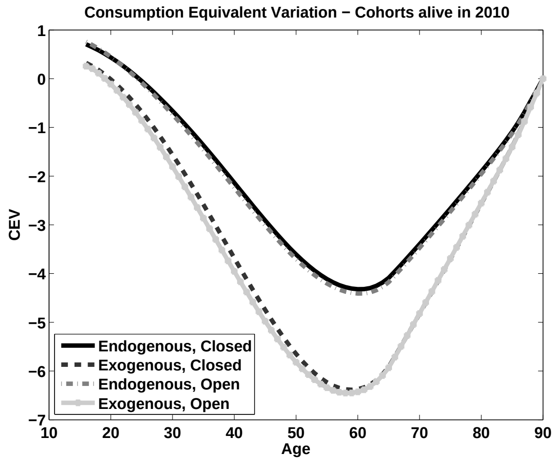
We summarize welfare consequences for newborns in table 2 and the maximum welfare losses in table 3. Again, the conclusion is that agents with the option to adjust their human capital investment will be relatively well off compared to agents with an exogenous efficiency profile and that this reaction can significantly contribute to dampen the welfare losses due to the price changes induced by aging. Note also that the different adjustment of the pension system has strong distributional effects. Firstly, the maximum welfare losses are much smaller for the model with constant replacement rates. Secondly, losses and gains are more evenly distributed.

Figure 9: Consumption Equivalent Variation of Agents alive in 2010, Benchmark Pension System

(a) Constant Replacement Rate



(b) Constant Contribution rate



Notes: "Endogenous" and "Exogenous" refer to the human capital production profile. "Open" and "Closed" refer to the results obtained from the closed and open economy versions.

Table 2: Welfare Gains / Losses - Newborns 2010

	Open Economy			
Pension System	Constant τ		Constant ρ	
	Endog.	Exog.	Endog.	Exog.
BM	0.8%	0.3%	-3.0%	-3.2%
PR	1.2%	0.6%	-1.7%	-2.3%

	Closed Economy			
Pension System	Constant τ		Constant ρ	
	Endog.	Exog.	Endog.	Exog.
BM	0.7%	0.3%	-2.8%	-3.1%
PR	0.9%	0.7%	-1.8%	-2.1%

Notes: "Endog." and "Exog." refer to the endogenous and exogenous human capital production profile. "Open" and "Closed" refer to the results obtained from the closed and open economy versions. "BM" denotes the benchmark pension system (constant retirement age), and "PR" denotes the pension reform (increase in the retirement age).

Table 3: Maximum Welfare Losses - Agents alive 2010

	Open Economy			
Pension System	Constant τ		Constant ρ	
	Endog.	Exog.	Endog.	Exog.
BM	-4.4%	-6.5%	-3.0%	-3.6%
PR	-3.6%	-6.0%	-1.9%	-3.0%

	Closed Economy			
Pension System	Constant τ		Constant ρ	
	Endog.	Exog.	Endog.	Exog.
BM	-4.3%	-6.4%	-2.8%	-3.1%
PR	-3.6%	-5.5%	-1.8%	-2.5%

Notes: "Endog." and "Exog." refer to the endogenous and exogenous human capital production profile. "Open" and "Closed" refer to the results obtained from the closed and open economy versions. "BM" denotes the benchmark pension system (constant retirement age), and "PR" denotes the pension reform (increase in the retirement age).

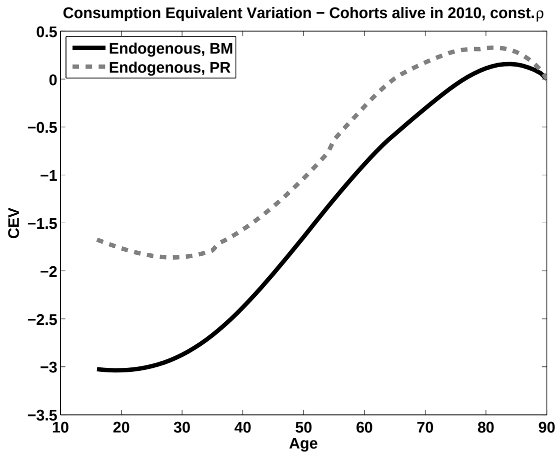
Welfare of Generations Alive in 2010 - Pension Reform

We next look at the differences in welfare changes for agents living in 2010 across all pension reform scenarios. As shown in figure 10 welfare losses in case replacement rates are held constant shrink to 1.9% for the youngest agents. Also in case of constant contribution rates, significant differences emerge. These are smaller because, as already seen from our analysis in section 4.1.1, the impact of increasing the retirement age is stronger in the constant replacement rate scenario.

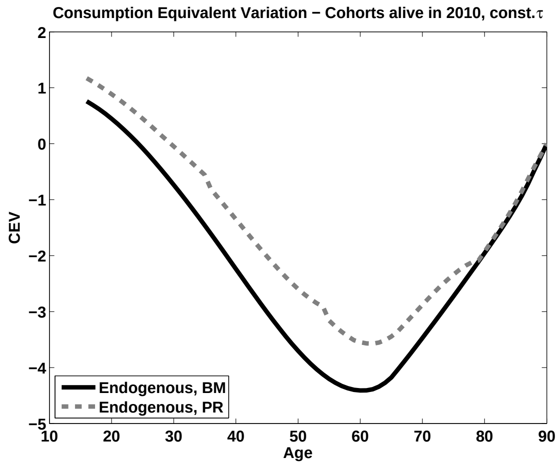
Maximum losses are summarized in table 3. Bringing all pieces together, maximum welfare losses are smallest – at $1.8 - 1.9\%$ – when the replacement rate is held constant and human capital can endogenously adjust.

Figure 10: Consumption Equivalent Variation of Agents alive in 2010, Pension Reform

(a) Constant Replacement Rate



(b) Constant Contribution Rate



Notes: All results obtained from the model with endogenous human capital and open capital markets.

Welfare of Future Generations - Benchmark Model

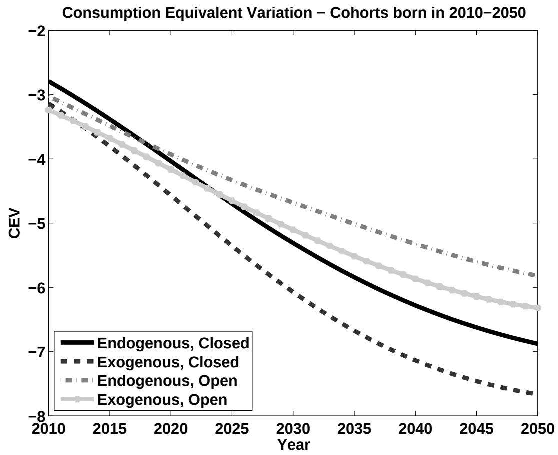
We now report welfare changes for newborns between 2010 and 2050. Holding the replacement rate constant has a strong negative impact on lifetime utility for future generations. Even with endogenous investment into human capital, maximum welfare losses can be up to 7% (5.9%) in the open (closed) economy. With an exogenous efficiency profile, losses are even higher. With constant contribution rates, newborns gain up to 2.5% (1.5%) in the open (closed) economy and endogenous human capital investment.

Welfare of Future Generations - Pension Reform

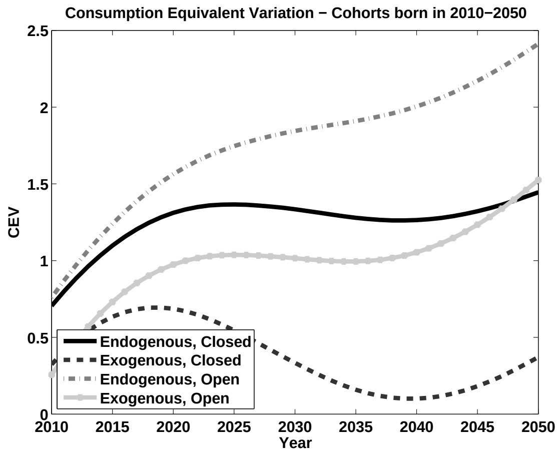
In the case of constant replacement rates, increasing the retirement age cannot totally eliminate welfare losses. For implementing the pension reform "PR" welfare still drops by 2.6% in our open economy scenario. With constant contribution rates, all future generations gain from demographic change. The largest gains are experienced by agents investing into their own human capital and living in an open economy.

Figure 11: Consumption Equivalent Variation of Agents born 2010-2050, Benchmark Pension System

(a) Constant Replacement Rate



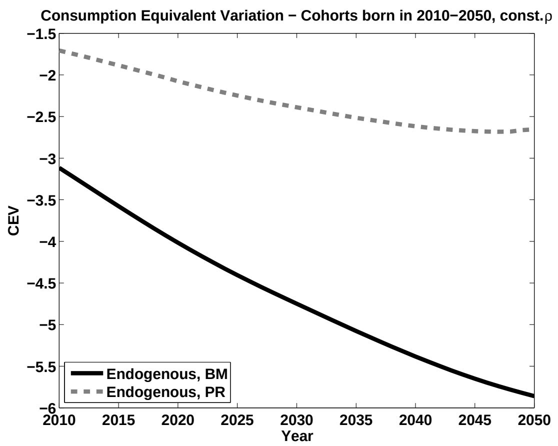
(b) Constant Contribution rate



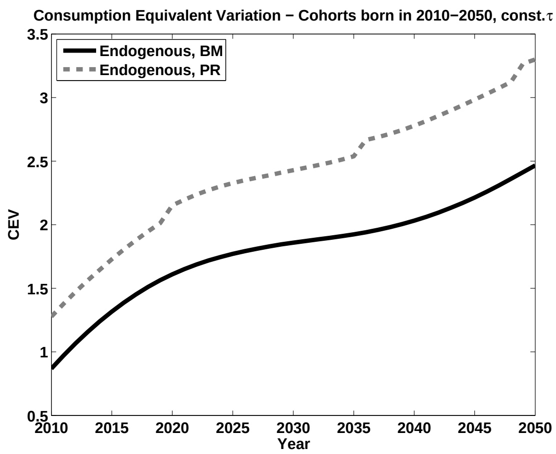
Notes: "Endogenous" and "Exogenous" refer to the human capital production profile. "Open" and "Closed" refer to the results obtained from the closed and open economy versions. "BM" denotes the benchmark pension system (constant retirement age), and "PR" denotes the pension reform (increase in the retirement age).

Figure 12: Consumption Equivalent Variation of Agents born 2010-2050, Pension Reform

(a) Constant Replacement Rate



(b) Constant Contribution Rate



Notes: All results obtained from the model with endogenous human capital and open capital markets.

5 Conclusion

This paper revisits the literature on the consequences of demographic change – aging – for welfare of generations who live through the demographic transition in industrialized countries by asking a straightforward question: How can the potentially detrimental consequences of aging be mitigated by two endogenous margins of adjustment: investing abroad and human capital formation? We address this question in combination with pension policy. That is, we ask how the design of pension policy may contribute to dampening via these endogenous channels.

We conclude with a negative and a positive finding. Our negative finding is that openness to international capital markets has only modest welfare implications vis-vis a counterfactual closed economy world. On the contrary, our positive finding is that endogenous human capital formation in combination with constant contribution rates to the pension system and increases of the statutory retirement age has strong welfare effects. If contribution rates are kept constant, then welfare of all future generations increases (strongly). Welfare losses for middle aged and asset rich households who suffer from decreasing returns on savings and decreasing pension payments are 6.5% if human capital cannot adjust and if the statutory retirement age is held constant. These losses decrease to 3.6% if human capital endogenously adjusts and increases of the retirement age roughly track life expectancy.

However, we have operated in a frictionless environment, where all endogenous human-capital adjustments are driven by relative price changes, increases in life-expectancy and increases of the statutory retirement age. If, instead, human capital formation is affected by market imperfections, such as borrowing constraints, then these automatic adjustments will be inhibited. In this case, appropriate education and training policies in aging societies in combination with pension policies are an important topic for future research and the policy agenda.

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A Computational Appendix

A.1 Household Problem

To simplify the description of the solution of the household model for given prices (wage and interest rate), transfers and social security payments, we focus on steady states and therefore drop the time index t and the country index i. Furthermore, we focus on a de-trended version of the household problem in which all variables x are transformed to $\tilde{x} = \frac{x}{A}$ where A is the technology level growing at the exogenous rate g. To simplify notation, we do not denote variables by the symbol $\tilde{\cdot}$ but assume that the transformation is understood. The de-trended version of the household problem is then given by

$$
\begin{aligned}
V(a,h,j) = \max_{c,\ell,e,a',h'} & \left\{ u(c, 1-\ell-e) + \beta s (1+g)^{\phi(1-\sigma)} V(a',h',j+1) \right\} \\
\text{s.t.} & \\
a' &= \frac{1}{1+g}\left((a+tr)(1+r)+y-c\right) \\
y &= \begin{cases} \ell h w (1-\tau) & \text{if } j < jr \\ p & \text{if } j \geq jr \end{cases} \\
h' &= g(h,e) \\
\ell &\in [0,1], \quad e \in [0,1].
\end{aligned} \tag{13}
$$

Here, $g(h,e)$ is the human capital technology.

Let $\tilde{\beta} = \beta s (1+g)^{\phi(1-\sigma)}$ be the transformation of the discount factor. Using the budget constraints, now rewrite the above as

$$
\begin{aligned}
V(a,h,j) = \max_{c,\ell,e,a',h'} & \left\{ u(c,1-\ell-e) + \tilde{\beta} V\left(\frac{1}{1+g}\left((a+tr)(1+r)+y-c\right), g(h,e), j+1 \right) \right\} \\
\text{s.t.} & \\
\ell &\geq 0.
\end{aligned}
$$

where we have also replaced the bounded support of time investment and leisure with a one-side constraint on ℓ because the upper constraints, $\ell = 1$, respectively $e = 1$, and the lower constraint, $e = 0$, are never binding due to Inada conditions on the utility function and the functional form of the human capital technology (see below). Denoting by μ_ℓ the Lagrange multiplier on the inequality constraint for ℓ, we can write the first-order conditions as

$$
c: \quad u_c - \tilde{\beta} \frac{1}{1+g} V_{a'}(a',h';j+1) = 0 \tag{14a}
$$

$$
\ell: \quad -u_{1-\ell-e} + \tilde{\beta} h w (1-\tau) \frac{1}{1+g} V_{a'}(a',h',j+1) + \mu_\ell = 0 \tag{14b}
$$

$$
e: \quad -u_{1-\ell-e} + \tilde{\beta} g_e V_{h'}(a',h',j+1) = 0 \tag{14c}
$$

and the envelope conditions read as

$$a: \quad V_a(a,h,j) = \tilde{\beta}\frac{1+r}{1+g}V_{a'}(a',h',j+1) \tag{15a}$$

$$h: \quad V_h(a,h,j) = \tilde{\beta}\left(\ell w(1-\tau)\frac{1}{1+g}V_{a'}(a',h',j+1) + g_h V_{h'}(a',h',j+1)\right). \tag{15b}$$

Note that for the retirement period, i.e. for $j \geq jr$, equations (14b) and (14c) are irrelevant and equation (15b) has to be replaced by

$$V_h(a,h,j) = \tilde{\beta} g_h V_{h'}(a',h',j+1).$$

From (14a) and (15a) we get

$$V_a = (1+r)u_c \tag{16}$$

and, using the above in (14a), the familiar inter-temporal Euler equation for consumption follows as

$$u_c = \tilde{\beta}\frac{1+r}{1+g}u_{c'}. \tag{17}$$

From (14a) and (14b) we get the familiar intra-temporal Euler equation for leisure,

$$u_{1-\ell-e} = hw(1-\tau)u_c + \mu_\ell. \tag{18}$$

From the human capital technology (3) we further have

$$g_e = \xi\psi(eh)^{\psi-1}h \tag{19a}$$

$$g_h = (1-\delta^h) + \xi\psi(eh)^{\psi-1}e. \tag{19b}$$

We loop backwards in j from $j = J-1,\ldots,0$ by taking an initial guess of $[c_J, h_J]$ as given and by initializing $V_{a'}(\cdot,J) = V_{h'}(\cdot,J) = 0$. During retirement, that is for all ages $j \geq jr$, our solution procedure is by standard backward shooting using the first-order conditions. However, during the period of human capital formation, that is for all ages $j < jr$, the first order conditions would not be sufficient if the problem is not a convex-programming problem. And thus, our backward shooting algorithm will not necessarily find the true solution. In fact this may be the case in human capital models such as ours because the effective wage rate is endogenous (it depends on the human capital investment decision). For a given initial guess $[c_J, h_J]$ we therefore first compute a solution via first-order conditions and then, for each age $j < jr$, we check whether this is the unique solution. As an additional check, we consider variations of initial guesses of $[c_J, h_J]$ on a large grid. In all of our scenarios we never found any multiplicities.

The details of our steps are as follows:

1. In each j, $h_{j+1}, V_{a'}(\cdot, j+1), V_{h'}(\cdot, j+1)$ are known.
2. Compute u_c from (14a).
3. For $j \geq jr$, compute h_j from (3) by setting $e_j = \ell_j = 0$ and by taking h_{j+1} as given and compute c_j directly from equation (23) below.

4. For $j < jr$:
   (a) Assume $\ell \in [0,1)$ so that $\mu_\ell = 0$.
   (b) Combine (3), (14b), (14c) and (19a) to compute h_j as

$$h_j = \frac{1}{1-\delta^h}\left(h_{j+1} - \xi\left(\frac{\xi\psi\frac{1}{1+g}V_{h'}(\cdot, j+1)}{\omega(1-\tau)V_{a'}(\cdot, j+1)}\right)^{\frac{\psi}{1-\psi}}\right) \tag{20}$$

   (c) Compute e from (3) as

$$e_j \quad = \tfrac{1}{h_j}\left(\tfrac{h_{j+1}-h_j(1-\delta^h)}{\xi}\right)^{\frac{1}{\psi}}. \tag{21}$$

   (d) Calculate $lcr_j = \frac{1-e_j-\ell_j}{c_j}$, the leisure to consumption ratio from (18) as follows: From our functional form assumption on utility marginal utilities are given by

$$u_c = \left(c^\phi(1-\ell-e)^{1-\phi}\right)^{-\sigma}\phi c^{\phi-1}(1-\ell-e)^{1-\phi}$$
$$u_{1-\ell-e} = \left(c^\phi(1-\ell-e)^{1-\phi}\right)^{-\sigma}(1-\phi)c^\phi(1-\ell-e)^{-\phi}$$

   hence we get from (18) the familiar equation:

$$\frac{u_{1-\ell-e}}{u_c} = hw(1-\tau) = \frac{1-\phi}{\phi}\frac{c}{1-\ell-e},$$

   and therefore:

$$lcr_j = \frac{1-e_j-\ell_j}{c_j} = \frac{1-\phi}{\phi}\frac{1}{hw(1-\tau)}. \tag{22}$$

   (e) Next compute c_j as follows. Notice first that one may also write marginal utility from consumption as

$$u_c = \phi c^{\phi(1-\sigma)-1}(1-\ell-e)^{(1-\sigma)(1-\phi)}. \tag{23}$$

   Using (22) in (23) we then get

$$\begin{aligned} u_c &= \phi c^{\phi(1-\sigma)-1}(lcr\cdot c)^{(1-\sigma)(1-\phi)} \\ &= \phi c^{-\sigma}\cdot lcr^{(1-\sigma)(1-\phi)}. \end{aligned} \tag{24}$$

   Since u_c is given from (14a), we can now compute c as

$$c_j = \left(\frac{u_{c_j}}{\phi\cdot lcr_j^{(1-\sigma)(1-\phi)}}\right)^{-\frac{1}{\sigma}}. \tag{25}$$

   (f) Given c_j, e_j compute labor, ℓ_j, as

$$\ell_j = 1 - lcr_j\cdot c_j - e_j.$$

   (g) If $\ell_j < 0$, set $\ell_j = 0$ and iterate on h_j as follows:

i. Guess h_j

ii. Compute e as in step 4c.

iii. Noticing that $\ell_j = 0$, update c_j from (23) as

$$c = \left(\frac{u_c}{\phi(1-e)^{(1-\sigma)(1-\phi)}} \right)^{\frac{1}{\phi(1-\sigma)-1}}.$$

iv. Compute μ_ℓ from (14b) as

$$\mu_\ell = u_{1-\ell-e} - \tilde{\beta} h w (1-\tau) V_{a'}(\cdot, j+1)$$

v. Finally, combining equations (14b), (14c) and (19a) gives the following distance function f

$$f = e - \left(\frac{\tilde{\beta} \xi \psi h^{\psi} \frac{1}{1+g} V_{h'}(\cdot)}{\tilde{\beta} \omega h (1-\tau) V_{a'}(\cdot) + \mu_\ell} \right)^{\frac{1}{1-\psi}}, \tag{26}$$

where e is given from step 4(g)ii. We solve for the root of f to get h_j by a non-linear solver iterating on steps 4(g)ii through 4(g)v until convergence.

5. Update as follows:
   (a) Update V_a using either (15a) or (16).
   (b) Update V_h using (15b).

Next, loop forward on the human capital technology (3) for given h_0 and $\{e_j\}_{j=0}^{J}$ to compute an update of h_J denoted by h_J^n. Compute the present discounted value of consumption, PVC, and, using the already computed values $\{h_j^n\}_{j=0}^{J}$, compute the present discounted value of income, PVI. Use the relationship

$$c_0^n = c_0 \cdot \frac{PVI}{PVC} \tag{27}$$

to form an update of initial consumption, c_0^n, and next use the Euler equations for consumption to form an update of c_J, denoted as c_J^n. Define the distance functions

$$g_1(c_J, h_J) = c_J - c_J^n \tag{28a}$$

$$g_2(c_J, h_J) = h_J - h_J^n. \tag{28b}$$

In our search for general equilibrium prices, constraints of the household model are occasionally binding. Therefore, solution of the system of equations in (28) using Newton based methods, e.g., Broyden's method, is instable. We solve this problem by a nested Brent algorithm, that is, we solve two nested univariate problems, an outer one for c_J and an inner one for h_J.

Check for uniqueness: Observe that our nested Brent algorithm assumes that the functions in (28) exhibit a unique root. As we adjust starting values $[c_J, h_J]$ with each outer loop iteration we thereby consider different points in a variable box of $[c_J, h_J]$ as starting values. For all of these combinations our procedure always converged. To systematically check whether we also always converge to the unique optimum, we fix, after convergence of the household problem, a large box around the previously computed $[c_J, h_J]$. Precisely, we choose as boundaries for this box $\pm 50\%$ of the solutions in the respective dimensions. For these alternative starting values we then check whether there is an additional solution to the system of equations (28). We never detected any such multiplicities.

A.2 The Aggregate Model

To solve the open economy general equilibrium transition path we proceed as follows: for a given $r \times 1$ vector $\vec{\Psi}$ of structural model parameters, we first solve for an "artificial" initial steady state in period $t = 0$ which gives initial distributions of assets and human capital. We thereby presume that households assume prices to remain constant for all periods $t \in \{0, \ldots, T\}$ and are then surprised by the actual price changes induced by the transitional dynamics. Next, we solve for the final steady state of our model which is reached in period T and supported by our demographic projections. For both steady states, we solve for the equilibrium of the aggregate model by iterating on the $m^c \times 1$ steady state vector $\vec{P}^c_{ss,j} = \left[p_{1,j}, \ldots, p_{m^c,j}\right]'$. $p_{1,j}$ is the capital intensity, $p_{2,j}$ are transfers (as a fraction of wages), $p_{3,j}$ are social security contribution (or replacement) rates and $p_{4,j}$ is the average human capital stock for region j. We perform this procedure separately for both world regions. Notice that all elements of $\vec{P}^c_{ss}$ are constant in the steady state.

To compute the open economy steady state we solve for the equilibrium of the aggregate model by iterating on the $m^o \times 1$ steady state vector $\vec{P}^o_{ss} = \left[p_1, \ldots, p_{m^o,j}\right]'$ where the number of parameters given by $m^o = (m^c - 1) + 1$. p_1 is the *common* capital intensity, $p_{2,j}$ are transfers (as a fraction of wages), $p_{3,j}$ are social security contribution (or replacement) rates and $p_{4,j}$ is the average human capital stock for region j.

Solution for the steady states for each closed region j (where we drop the region index for brevity) of the model involves the following steps:

1. In iteration q for a guess of $\vec{P}^{c,q}_{ss}$ solve the household problem.
2. Update variables in $\vec{P}^c_{ss}$ as follows:
   (a) Aggregate across households to get aggregate assets and aggregate labor supply to form an update of the capital intensity, p^n_1.
   (b) Calculate an update of bequests to get p^n_2.
   (c) Using the update of labor supply, update social security contribution (or replacement) rates to get p^n_3.
   (d) Use labor supply and human capital decisions to form an update of the average human capital stock, p^n_4.
3. Collect the updated variables in $\vec{P}^{c,n}_{ss}$ and notice that $\vec{P}^{c,n}_{ss} = H(\vec{P}^c_{ss})$ where H is a vector-valued non-linear function.
4. Define the root-finding problem $G(\vec{P}^c_{ss}) = \vec{P}^c_{ss} - H(\vec{P}^c_{ss})$ and iterate on $\vec{P}^c_{ss}$ until convergence. We use Broyden's method to solve the problem and denote the final approximate Jacobi matrix by B_{ss}.

Solution for the steady states of the open economy of the model involves the following steps:

1. In iteration q for a guess of $\vec{P}^{o,q}_{ss}$ solve the household problem.
2. Update variables in $\vec{P}^o_{ss}$ as follows:
   (a) Use the guess for the global capital intensity to compute the capital stock for region j compatible with the open economy, perfect competition setup. Use this aggregate capital stock with the aggregate labor supply to form an update of the global capital intensity, p^n_1.

(b) Calculate an update of bequests to get $p^n_{2,j}\ \forall j$.

(c) Using the update of labor supply, update social security contribution (or replacement) rates to get $p^n_{3,j}\ \forall j$.

(d) Use labor supply and human capital decisions to form an update of the average human capital stock, $p^n_4\ \forall j$.

3. Collect the updated variables in $\vec{P}^{o,n}_{ss}$ and notice that $\vec{P}^{o,n}_{ss} = H(\vec{P}^o_{ss})$ where H is a vector-valued non-linear function.
4. Define the root-finding problem $G(\vec{P}^o_{ss}) = \vec{P}^o_{ss} - H(\vec{P}^o_{ss})$ and iterate on $\vec{P}^o_{ss}$ until convergence. We use Broyden's method to solve the problem and denote the final approximate Jacobi matrix by B_{ss}.

Next, we solve for the transitional dynamics for the closed economies by the following steps:

1. Use the steady state solutions to form a linear interpolation to get the starting values for the $m^c(T-2)\times 1$ vector of equilibrium prices, $\vec{P}^c = \left[\vec{p}_1', \dots, \vec{p}_m^{c,\prime}\right]'$, where $p_i, i = 1, \dots, m^c$ are vectors of length $(T-2)\times 1$.
2. In iteration q for guess $\vec{P}^{c,q}$ solve the household problem. We do so by iterating backwards in time for $t = T-1, \dots, 2$ to get the decision rules and forward for $t = 2, \dots, T-1$ for aggregation.
3. Update variables as in the steady state solutions and denote by $\tilde{\vec{P}}^c = H(\vec{P}^c)$ the $m^c(T-2)\times 1$ vector of updated variables.
4. Define the root-finding problem as $G(\vec{P}^c) = \vec{P}^c - H(\vec{P}^c)$. Since T is large, this problem is substantially larger than the steady state root-finding problem and we use the Gauss-Seidel-Quasi-Newton algorithm suggested in Ludwig (2007) to form and update guesses of an approximate Jacobi matrix of the system of $m^c(T-2)$ non-linear equations. We initialize these loops by using a scaled up version of B_{ss}.

We then solve for the transitional dynamics for the open economies by the following steps:

1. Use the equilibrium transition solutions from the closed economies to get the starting values for the $m^o(T-\tilde{t}-2)\times 1$ vector of equilibrium prices, $\vec{P}^o = \left[\vec{p}_1', \dots, \vec{p}_{j,m}^{o,\prime}\right]'$, where $p_i, i = 1, \dots, m^o$ are vectors of length $(T-\tilde{t}-2)\times 1$ where $\tilde{t}$ is the year of opening up.
2. In iteration q for guess $\vec{P}^{o,q}$ solve the household problem. We do so by iterating backwards in time for $t = T-\tilde{t}-1, \dots, 2$ to get the decision rules and forward for $t = 2, \dots, T-\tilde{t}-1$ for aggregation where $\tilde{t}$ denotes the year of the opening up. For agents living in year $\tilde{t}$ we store the distribution of assets, human capital, and population in year $\tilde{t}$ and solve the household problem for all households only for their remaining lifetime.
3. The we proceed then as in the case for the closed economies (updating) but define the root-finding problem now for the open economy as $G(\vec{P}^o) = \vec{P}^o - H(\vec{P}^o)$ which we solve by the same method as above.